Breakdown of a Relativist Myth: Mass Never Depended on Velocity.

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I

It is a most basic tenet, or dogma, of modern physics (special relativity) that the mass of moving bodies should depend on the velocity of their motion. This had been unthinkable on the basis of the Galileian-Newtonian theory of motion, where "mass" (as a shorthand term for "quantity of matter" introduced by Newton) synonymously stood for a certain *quantity of matter* as a sum of microscopic material particles ("atoms" in the sense of the Ancients) that should be present in a certain macroscopic body. Based on, and derived from experimental experience, macroscopic bodies were understood to consist of this material quantity, and of void filling the interstices between these elementary particles, so that "matter" and "void" together should constitute the macroscopic appearance of a body. Anti-atomists, say the followers of Aristotle in older and Descartes in newer times, denied this view. One of them was Ernst Mach who, being a *dogmatic* anti-atomist, wanted to understand "mass" not in Newton's sense as a *quantity of elementary particles,* but as a *quality* of matter, based on the idea that matter should be an unstructured continuum, and macroscopic bodies should consist of this continuous matter only. Matter, then, or bodies, were supposed "to have mass" as one of the *qualities of matter* from then on. This semantic change at pleasure of the term "mass" enabled Mach to suspect as a physical possibility that a body's "mass" might depend on its velocity of motion. In 1883 he wrote somewhat prophetically that the belief of mass to be a physical *invariable* might be shaken in the future similarly as the idea of an invariable quantity of heat had been shaken at that time [1]. The suspicion seemingly became certain with experiments performed in order to measure the mass of electrons moving in cathode-ray tubes, around the years 1901/02 (Walter Kaufmann [2]).

II

On closer inspection of Kaufmann's reports on his experiments, one finds him relying on the electron theory of the theorist M. Abraham, as published in 1901 [3]. Abraham, vice versa, draws explicitly on Kaufmann's "höchst interessantes", i.e. *most interesting,* allegedly *by experiment achieved result*, according to which the "mass" or *inertia* of electrons should *really depend on* their velocity of motion [4]. On this basis, Abraham aims at answering the following questions: "Ist es möglich, diese Abhängigkeit quantitativ aus den Differentialgleichungen des electromagnetischen Feldes abzuleiten? Ist die Trägheit des Electrons vollständig durch die dynamische Wirkung seines electromagnetischen Feldes zu erklären, ohne eine von der electrischen Ladung unabhängige Masse zu Hilfe zu nehmen?" (in Engl.: *Is it possible to explain the said dependence quantitatively by deriving it from the differential equations of the electromagnetic field? Is the inertia of the electron completely explainable as a dynamic effect of its electromagnetic field without resorting to a mass that does not depend on the electric charge?*).

Abraham, no wonder, succeeded. Let us see how he achieved his goal mathematically.

1. First, simply presupposing the existence of some 'electromagnetic mass' besides 'material mass', he freely defines this *electromagnetic mass,* based on d'Alembert's principle [5], as an additional term m (besides the *material mass* M) of the basic equation

$$(M + m)\, dq/dt = K; \tag{1}$$

q stands for "velocity", of course. In the following I shall replace q by the letter v.

2. Next, Abraham defines his problem as the task how to calculate the electromagnetic mass m, provided one knows the field that corresponds to a uniform motion of electrons. For this, he continues, one must derive from the field equations such integrals of the equations of motion which contain only the term of velocity, v.

In the course of the calculation Abraham, explicitly referring to a so-called "Reduktionsverfahren" developed by H.A. Lorentz and H.F.C. Searle, calculates the convection potential of the moving electron by projecting via transformation of coordinates the whole moving system S onto a system S' of electric charges at rest: The relative velocity of S with reference to S' (which represents the electron's velocity of motion in the system S) enters into the transformation of the x-coordinate according to $x = v/c$ (with c = vacuum velocity of light):

$$x' = x/(1 - x^2)^{1/2} \quad [6] \tag{2}$$

As a consequence of the quotient *v/c*, since the dimensions of *v* and *c* cancel each other, the velocity of motion of the electron *vanishes* into a simple numerical factor, *x*.

3. It is this dimensionless variable factor *x* then which, instead of the wanted velocity *v*, enters into Abraham's final equations for the determination of the *electromagnetic mass* [7]. For its transversal component he obtains

$$m_r = - 1/c^2 \times 1/x \times dU/dx \tag{3}$$

for its longitudinal component he obtains

$$m_s = - 1/c^2 \times d^2U/dx^2 \tag{4}$$

where *U* is used to denote the electrostatic energy [8]. So I replace it by the variable *E* that shall absorb the dimensionless terms without physical meaning of eqs. (3,4), which terms imply the velocity *v* of the electron's motion (but only as part of the quotient *v/c*, i.e. decomposed into a mere variable number). Then, by means of dimensional analysis we can see both equations to result in the calculation of mass $m_{r,s}$ according to the paradigm

$$m = E/c^2 \ . \tag{5}$$

It's nice, by the way, to find not only the method of coordinate transformation, but also the so important "Einstein equation" already here, three years before Albert Einstein' famous paper of 1905 on special relativity (which, of course, doesn't mention Abraham's electron theory).

Eq. (5) then shows the desired result that the mass of the electron *m* obviously must vary in proportion to the variable *E*, since the vacuum velocity of light *c* is constant. And this result is nothing but the consequence of a funny mathematical idea, namely to represent the velocity of a moving object through the velocity of motion of its frame of reference relatively to a like system at rest, in the course of which the velocity *v* of the moving electron becomes replaced by a mere numerical variable according to the dimensionless quotient *v/c.*

III

The instance shows the way of reasoning of theoretical physics that is especially characteristic of Einstein's establishing special relativity, and its most important result, the so-called "mass-energy equivalence" (which is not an equivalence of mass and energy at all, as dimensional analysis easily shows [9]). It is the way of *first* to develop, on the ground of a hypothesis such as 'suppose mass to be a variable', a corresponding mathematical formalism, *and then*, from the fact that this formalism seemingly corroborates the hypothesis, *to infer* by deduction from it *that nature must really behave* as the said formalism describes it [10]. In a final step, then, an accordingly interpreted experiment will be understood as the 'experimental proof' of the presupposed theory, which from then on is meant to immunize the theory against any criticism. As Einstein once put it, in a famous discussion with Werner Heisenberg: *It is only the theory* [i.e. the hypothesis in its mathematical shape] *that tells about what one can observe in nature* [11]. Of course this is often only a circular argument without any probative force, and the method is always quite the contrary of Newton's device "hypotheses non fingo". Newton was convinced that the hypothetical-deductive method could at best produce fairy-tales about nature. The instance of "mass depending on velocity" proves him right.

Behind eq. (5) there is, however, certainly some kind of truth. As it is a representation of Poynting's 1884 derivation of the energy-momentum proportionality $E/p = c$, we may replace the "p" by its equivalent mv, i.e. the product of mass m and velocity v of the electron. Then the resulting equation

$$E = (mv) \times c \tag{6}$$

clearly might show that of course the mass m of a moving electron doesn't vary with its velocity, since eq. (6) exhibits the proportional variables E and v, while m remains an invariant, and c the constant factor of proportionality. Eq. (6), by the way, is the true result of Einstein's well-known attempts to derive "his" equation $E = mc^2$ through "classical" operations only (for this see Max Born [12]).

References:

[1] Ernst Mach, Die Mechanik in ihrer Entwicklung, 7. Aufl. (1912) Leipzig, p. 215.

[2] W. Kaufmann, Göttinger Nachrichten Heft 2 (1901) pp.143-155; Heft 2 (1902) pp. 291/6.

[3] M. Abraham, Göttinger Nachrichten Heft 1 (1902) pp. 20-41.

[4] M. Abraham l.c. p. 21.

[5] l.c. p. 22, eq. (2); p. 24, eq. (3).

[6] l.c. p. 32 eq. (19).

[7] l.c. p. 38 eq. (25c).

[8] l.c. p. 38 eqs. (25d, 25e).

[9] Max Jammer, Concepts of Mass in Contemporary Physics and Philosophy, (2000) Princeton University Press, p. 89.

[10] Bernulf Kanitscheider, Das Weltbild Albert Einsteins, (1988) München, p. 83.

[11] Werner Heisenberg, Der Teil und das Ganze, 8. Aufl. (1984) München, p. 80.

[12] Max Born, Die Relativitätstheorie Einsteins, 5. Aufl. (1969) Berlin (Springer), p. 245.

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