A Comment on the 1999 Cohen-Whitman Edition of Newton's *Principia.*

Preliminary note.

In the year 1999, there appeared a new English edition of Newton's *Principia*, edited by I. Bernard Cohen and Anne Whitman, preceded by "A Guide to Newton's *Principia"* by I. Bernard Cohen. In the "Preface" to his "Guide", Cohen invites the reader "to send to our publisher any suggestions for the improvement of our text as well as corrections of errors of fact or interpretation in the Guide." So in the year 2000 I sent four letters to The University of California Press, which I present in the following. To none of these letters did I ever receive an answer.

Ed Dellian D-14169 Berlin, Aug. 8, 2000

The University of California Press
Berkeley 94720, California
U.S.A.

ISAAC NEWTON, THE PRINCIPIA
A new Translation by I. Bernard Cohen and Anne Whitman
Preceded by A Guide to Newton's *Principia* by I. Bernard Cohen

Dear Sir/Madam,

as a reader of your 1999 edition of Newton's *Principia* and of I. Bernard Cohen's „Guide",
I want to follow the advice of Prof. Cohen on P. XV of his „Preface" which reads as follows:
„Readers are requested to send to our publisher any suggestions for the improvement of our text as well as corrections of errors of fact or interpretation in the Guide."

I take the liberty of suggesting some corrections of errors of fact and interpretation to be found in Prof. Cohen's „Guide".

1. On p. 99 Prof. Cohen aims to explain Newton's concept of a „force of inertia", stating that

> *„Numerous examples come to mind to illustrate what Newton means when he says that this 'force' is exerted by a body only when the body resists a change in state. Consider a heavy truck moving on horizontal rails (so that friction is minimized). A great effort is required in order to get the truck moving from rest (as if one is overcoming a force of resistance to a change in state), but it takes relatively little effort to keep the truck moving. Again, to bring*

the moving truck to a halt requires an effort that makes it seem as if there is a force to be overcome. A similar example would be a forward-moving projectile that hits a wall, at which point a force is exerted which may be great enough to make a hole in the wall".

The whole paragraph is mistaken and should therefore be cancelled in further editions.

(1) Cohen here ignores the fact that the *Principia*, Book I, deals with *„motion in free spaces, that is, spaces devoid of resistance*", as he himself states it on p. 128. Both examples, the truck as well as the projectile, belong to the motion of bodies that are externally resisted, the truck by friction with air etc., the projectile by the resistance of the wall. And that is the concern not of Book I, but of Book II of the *Principia.* This is true with respect to the truck even if *„friction is minimized*". The then remaining *„relatively little effort to keep the truck moving"* is due not to the innate „force of inertia" which Prof. Cohen aims to illustrate, but to the still remaining external friction only: If there remained no friction at all, the truck, once set in motion, according to Newton's first law of motion would forever *„persevere in its state of moving uniformly straight forward"* (*Principia* p. 416) without needing a n y additional effort to keep moving.

(2) Cohen also contradicts Newton's second law, in so far as he assumes that *„a great effort is required in order to get the truck moving from rest"*. In fact, according to the second law such an effort is never „great" nor is it „little", rather it is always p r o p o r t i o n a l to the generated respective motion of the truck. Consequently even the smallest „effort" or force will cause a proportional motion of the heaviest truck - remember in spaces devoid of resistance, which is the *general* presupposition of the theory of motion as taught in the *Principia*, Book I. And the effort that is required to change the motion of the truck as *„to bring the moving truck to a halt"*, does not only *„make it seem as if there is a force to be overcome"*, rather it shows that the truck really „*exerts* [the force of inertia] *during a change of its state caused by another force* [i.e. Cohen's „effort"] *impressed upon it"*, as Newton says it, in his definition 3 on p. 404. The *exerted* inertial force, however, is no longer an *inner* force ("materiae vis insita", as Newton calls it), but an *external one, as it is "exerted"; and it is of course that force that the moving body, if slowed down, impresses on the obstacle.* And the effort or force that is required to bring the truck to a halt again is neither „great" nor „little", rather it is *always proportional* to the quantity of change in the motion of the truck that is required to make it come to a halt, according to Newton's second law. Now, if this force sometimes should seem *relatively great* even though the truck moves very slowly, this is due to the fact that not the velocity only, but the quantity of „motion" of the truck has to be overcome, which quantity results from the product of the mass of the truck with its velocity, according to the *Principia*, definition 2. A very heavy truck that moves very slowly will nevertheless represent a sometimes surprisingly great quantity of motion „mass times velocity", generated by a relatively great quantity of impressed force proportional to that generating motion.

(3) Prof. Cohen on p. 99 aims at explaining *„what Newton means when he says that this 'force'* [i.e. the force of inertia] *is exerted by a body only when the body resists a change in state."* Again Cohen offers a misinterpretation. Evidently he refers to *Principia*, definition 3 (p. 404 of his book), where Newton writes: *„A body exerts this force only during a change of its state, caused by another force impressed upon it..."*. But the case under discussion here is not that [due to the exertion of the force of inertia] *„the body resists a change in state"*, as Cohen puts it. Newton rather indicates that the body exerts this „innate" force *„during a change of its state"*, that is: The „innate" force *turns from internal and passive to external and active,* whilst, and so long as, the body, according to the second law, in time and space

performs the change in state that is proportional to *„another force impressed on it“*. Now, since according to the said second law, the performed change in the motion of a body should a l w a y s be proportional to the impressed force, this change in motion will not be diminished by any „resistance“ of the body, as Cohen suggests it. So Newton by this inertial force means just the evident faculty of material bodies *to interact* with external forces impressed on them, i.e. their innate force to react on such impressions *by changing their state of rest or motion.* Imagine a body devoid of such a force, and you will see that this body would *not at all* be able to interact with any such external forces, and consequently would never change its state. After all, the „force of inertia“ designates a force of „resisting“ which should never be confounded with „resistance“ in the common sense. Newton consequently states in def. 3 and 4 that this force can as well be seen as „impetus“, i.e. as a force innate in matter *to causally maintain the uniform-straight-lined motion of the body*. This force is passive and internal so long as the body moves uniformly in a straight line, but becomes active, and is „exerted“ in the *interaction* with an external acting force, so long as this external force is impressed on the body to change its state. Which force of inertia is unknown in the so-called classical or „Newtonian“ mechanics, where the faculty of bodies to interact with external forces is simply conceived as a *property* of *matter itself*, and where the interaction always happens *instantaneously*, so that there is no spacetime stage of gradual interaction, and no time at all for a substantial „force of inertia“ of its own ontological status to be „exerted“.

(4) As a consequence, I must point to a further error of Prof. Cohen. It is at least misleading to state that *„Newton's concept* [of a force of inertia] *with the elimination of the 'force', became the standard of classical physics“* (as Prof. Cohen puts it on p. 101). Actually, Newton's concept „with the elimination of the 'force' “, is no more Newton's concept of a „force of inertia“ as a real physical entity of its own, rather it is *reduced to the materialist concept of a property of matter itself*, which certainly contradicts Newton's philosophy of force and motion. Indeed this reduction, as for instance explicitly claimed and performed by Immanuel Kant (!) in 1786, in his „Metaphysische Anfangsgründe der Naturwissenschaft“, is (only a little) part of the corruption which materialists of the 18th century did to Newton's theory, in order to reduce it to a materialist theory of matter in motion only, excluding immaterial generating „forces“ (active and passive). Note that even the force of gravity, as well as the force of inertia, for nowadays physics is *a property of matter only*. Now, if one lets pass this materialist reduction so carelessly as Prof. Cohen does it, one will certainly miss the point of Newton's philosophy of nature, which necessarily includes the concepts of external and internal, active and passive „forces“ that hold an ontological (immaterial) status of their own, to cause in material bodies (actively) the change of states of motion or rest, and (passively) the perseverance of such states as long as active forces are absent. In fact, Newton's theory of force and motion differs fundamentally from the materialist „classical mechanics“ that knows only one single, always accelerating „force“, which concept, as it is *identified* with its material effect on the motion of bodies according to the formula „force equals mass-acceleration“, *materializes the „force“ to accelerated motion only*, thus destroying the Newtonian *dualism* of immaterial generating forces („active principles“, as Newton sometimes said) and their material effects. To call the result still 'Newtonian mechanics' (as Prof. Cohen implicitly does it on p. 101 line 17/8) is the more intolerable as the philosophical principles lying behind all stem from Newton's philosophical antipodes G.W. Leibniz and René Descartes, to mention only the Leibnizian principle „causa aequat effectum“ (instead of the proportionality of cause and effect as taught by Newton, in the *Principia*, second law). Unfortunately, Prof. Cohen, pretending to guide the reader to the understanding of Newton's „mathematic principles of natural philosophy“, in fact *avoids any* discussion of the *p h i l o-*

s o p h i c a l principles which Newton demonstrates mathematically in his *magnum opus* (cf. p. 96 line 26/7, p. 98 line 11/2 of the „Guide“). One consequence of this abstinence can be seen in the issue of the term „impetus“ used by Newton not only in the def. 3 of the „force of inertia“. This Latin term „impetus“ had been erroneously translated by Andrew Motte into „impulse“. Prof. Cohen tacitly returns to the Latin „impetus“ - not saying a word, however, about the far-reaching philosophical implications of this alteration, and about the meaning of this special term which is totally unknown to nowadays readers. Altogether, Prof. Cohen's „Guide“ wants to interpret the *Principi*a as a work not in philosophy, but in physics. How imperfect this point of view is, would clearly be revealed if Prof. Cohen had added an *index* to his „Guide“, which the reader badly misses, judging its absence as an-other shortcoming of Cohen's work.

2. Prof. Cohen's indifference as to the real meaning of *Newton's philosophy* becomes evident again in his discussion of Newton's second law on p. 92, p. 104, pp. 110. Confronted with the problem that the wording of the second law does not at all fit with its usual interpretation „force equals mass-acceleration“, Prof. Cohen tries to blur the difference by bluntly alleging that Newton „*in the Principia sets forth a dimensionless physics*“ (p. 92 line 22). As to what extent this most surprising argument is intolerable and wrong, I have tried to show in the enclosed paper. - In this context, another mathematical mistake of Cohen must be criticized. On p. 92 line 24-6, after having introduced two seemingly possible different interpretations of the second law, Cohen writes*: „The modern reader would be troubled by this example because the constant of proportionality in the two cases must be of a different dimensionality.*“ This statement is wrong, because the said constant of proportionality must n o t be of different dimensionality if the two introduced concepts of „force“ sensibly refer to *different forces* (one „impulsive“, the other „continuous“, as Cohen himself presupposes on pp. 112-3).

Sincerely yours

Ed Dellian

I enclose a paper „*Mutationem motus proportionalem esse vi motrici impressae*“ or: How to understand Newton's second law of motion, after all“ (draft, Aug 8, 2000).

Ed Dellian D-14169 Berlin, Aug. 29, 2000

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ISAAC NEWTON, THE PRINCIPIA
A new Translation by I. Bernard Cohen and Anne Whitman
preceded by a Guide to Newton's *Principia* by I. Bernard Cohen

Dear Sir/Madam,

hereby I shall continue what I have set in with my letter of Aug. 8, 2000. In the following, you will find in 33 points a list of errors, mistakes, misinterpretations and corruptions of Newton's authentic theory, as they unfortunately are to be found in Prof. I. Bernard Cohen's „Guide". As a matter of fact, these errors etc. are not only those of Prof. Cohen, rather they result from 300 years of efforts „to move Newton's text from being a work in philosophy toward being the foundation for modern science" (Margaret C. Jacob), respectively for "physics", which process has also been called a „positivistic interpretation" (Paolo Casini), but much more should be seen (according to Betty J. T. Dobbs) as a *materialist corruption* of Newton's authentic teaching. It is clear that all these things ought to come to light in a „Guide" to Newton's *Principia* which compares Newton's original text with principles of nowadays „classical" mechanics, as Prof. Cohen does it. Prof. Cohen, however, is not aware of *any* errors or misinterpretations etc., rather he joins the said efforts by varnishing and suppressing inconsistencies wherever they might appear, and thus he maintains the historical *proton pseudos* to obscure the *Principia* as if it were only a foundation of mechanics, and „classical mechanics" were Newton's legitimate daughter. The following list should warn the readers of Cohen's „Guide" against relying too much on it. I shall start from chapter 1 (page 11) of the „Guide", and go to p. 108, to cover chapters 1-4.

1. On p. 11, Cohen opens his work and his attitude toward Newton stating that

> „Leibniz introduced the name 'dynamics'. Although Newton objected to this name, 'dynamics' provides an appropriate designation of the subject matter of the *Principia*...".

The term „dynamics" was indeed invented by Newton's philosophical antipode G.W. Leibniz to christen his own theory of motion (Specimen Dynamicum, Leipzig 1695) as an *antidote* to Newton's *Principia* of 1687. Moreover, it is a term that explicitly refers to that Aristotelian theory of 'force' and 'motion' as *intrinsic properties of matter* which Galileo had begun to overthrow, against which Newton had expanded Galileo's teaching to a complete causal theory of force as 'cause', and motion as its proportional 'effect', and which Descartes and Leibniz, contrary to Galileo and Newton, had aimed to *restore*, by only improving it a little according to some experimental findings such as Galileo's law of fall. This Aristotelian theory, and certainly its modernized Leibnizian form christened 'dynamics'as well, is the goal of Roger Cotes's harsh criticism, in his preface to the second edition of the *Principia* (1713),

cf. Cohen-Whitman p. 385, paragraph two, and thus it should never be confused with Newton's philosophy.

As to its contents, 'dynamics' in the Aristotelian language as well as in its modern sense, does not distinguish between *unresisted motion in free space* (which is the contents of the *Principia*, book 1) on the one hand, and *resisted* motion on the other, i.e. motion against the resistance of friction, e.g. of air, cohesion *etc.(which is a contents of the Principia*, b*ook 2).* In fact it deals, as a foundation of *applied* mechanics, *exclusively* with *resisted* motion in e.g. *hydrodynamics* and *thermodynamics* .For Newton's teaching, however, the foundation of a *philosophical* theory of force and motion on *philosophical* concepts of force and motion, time and space, to be found in book 1 as a highly abstract theory of *unresisted motion in a void*, is basic. Such a theory was never acceptable for the 'Aristotelians' Descartes and Leibniz who rejected the possibility of a void, and consequently the reasonability of a theory of unresisted motion as well. For these reasons, Leibniz would never have allowed the Newtonians to use his term 'dynamics', nor would Newton ever have used this term for his theory of motion. Consequently, Cohen *erroneously* states in his footnote on p. 11 that Newton's objections to the name 'dynamics' „were not based on (the name's) adequacy or inadequacy to describe the subject matter."

Indeed, he who wants to understand the basic concepts of Newton's theory (which are presented and explained by Newton in the *Principia*, book 1) according to Cohen's guiding suggestion as if this theory could be understood as 'dynamics', will certainly miss the point of it; and so does Cohen himself. This my conclusion is clearly corroborated by the erroneous way in which he tries to explain „what Newton means" with e.g. the concept of „vis insita" (*Principia*, book 1, def. 3), giving some examples on p. 99 which belong clearly to the subject matter 'resisted motion' of *Principia*, book 2. I have already pointed out this mistake of Cohen in detail, in my letter of Aug. 8.

Throughout his „Guide" to the *Principi*a, book 1, Cohen, in order to avoid giving reverence to Newton's terms „natural philosophy" or „experimental philosophy", instead makes general use of the Aristotelian/Leibnizian term „dynamics" to designate Newton's theory, even forming the phrase „Newtonian dynamics" (e.g. on p. 14). This mode gives certainly a striking expression of Cohen's independent attitude as to Newton's true intentions, feelings and objections as well as of his acquaintance with the main difference between the Newtonian and the Aristotelian/Leibnizian theories of motion.

2. On p. 13, Cohen states that „book 2 of the *Principia* contains an expanded version of the analysis of motion in resisting mediums".

Actually book 2, to *clearly contrast* the theory of *absolutely unresisted motion* of book 1, opens in sect. 1-3 with a *general* theory of motion against *any* sort of resistance, for which resistance that of a more or less dense material medium is *only one example.* In order to make really clear the difference between book 1 and book 2, as explained above, Cohen should have said that book 2 contains a version of the analysis 'of motion of bodies against *any* resistance *whatever'*.

3. On p. 19, Cohen states that

„mass is not a primary variable in Newton's mode of developing his dynamics [!] in book 1. In fact, most of book 1 deals exclusively with particles. Physical bodies with significant dimensions or shapes do not appear until sec. 12."

Contrary to this statement, 'mass' *is of course* a primary variable *throughout* the *Principia*; and so says Cohen himself on p. 93, line 5-6. 'Mass' is a primary variable *not only* in the opening def. 1, where this term, equivalent to the „quantity of matter" in a body, is said to mean (in Newton's use) the same as the term „body", and to be proportional to (i.e. to vary in proportion to) the body's weight. In def. 1, Newton explicitly says: „Furthermore, *I mean this quantity (of matter)* whenever I use the term 'body' or 'mass' in the following pages" (Cohen-Whitman p. 404, line 1-2; my italics). Now, since the term 'quantity of matter', 'body', or 'mass' appears in def. 2,3,4,5,7,8, in the following *Scholium*, in the first, second and third law of motion, and in all the following up to sect. 12 *abundantly*, how dares Cohen say that „most of book 1 deals exclusively with (single) particles"? The point is that Newton often explains different motions of *one and the same body* under the influence of different forces. In this case, the moving quantity of matter is *always one and the same*, and consequently *it must not be considered as a variable* in considering the motion of the body. It must, however, be considered in *any* cases of motion as a *constituent* of that motion, since 'motion' is generally defined by Newton through the product 'quantity of matter times velocity' *(Principia*, def. 2; Cohen-Whitman p. 404), so that e.g. double the force in a double body according to Newton's second law generates *double the motion*, even if *the velocity* of the double body remains the same as that of the single body (as it is e.g. the case with gravitational force and motion, i.e. the well-known case of equal velocity of free fall of different bodies).

4. On p. 32, Cohen correctly criticizes the „fault to modernize Newton's thought" through the „elimination of the word 'force'." However, in this context the reader misses badly an information about the *meaning* of the word „force". It makes little sense if Cohen, *as a translator*, here insists on translating e.g. „vis inertiae" into „force of inertia" in order to „present Newton's thought", whilst he, *as writer of a „Guide" to Newton*, wants to make us believe that this very term is quite useless for an understanding of the *Principia*, and may be omitted e.g. in the case of „vis inertiae" in favor of only „inertia" as a *property of matter* (p. 84 below, 101 paragraph 2).

In fact, the manner to explain phenomena (such as the „inertial motion" of a body with constant velocity in a straight line *ad infinitum*, according to the first law of motion) either as *properties of matter*, or as *effects of generating „forces", marks a main difference* between a materialist philosophy, which Newton abhorred, and Newton's own philosophy of nature. Clearly this insight requires the understanding of Newton's 'force' as a generating or persevering 'cause' of motion, and inevitably also includes the view of such natural causes to be real entities of an ontological state independent from matter, i.e. of an *immaterial state*. The concern is addressed in Roger Cotes's preface to the *Principia's* second edition (Cohen-Whitman p. 385), where the idea to explain natural phenomena simply by properties of matter (e.g. the falling of bodies to the earth by their proper 'heaviness') is attributed to „the whole of Scholastic doctrine derived from Aristotle and the Peripatetics". Obviously, then, Newton's alternative rests decisively on the meaning of 'force' as an immaterial thing, and the reduction of Newtonian forces to properties of matter (inertness, heaviness) that happened during the 18th and 19th century can be understood as a process to reduce Newton's teaching to a materialist theory of only body and motion, much in the sense of René Descartes and his followers. It is true that in modern 'Newtonian mechanics' the term 'force' is nothing more than a word which mostly designates not force as a *cause*, but rather *motion as its effect*, say

accelerated motion, according to the formula „force equals mass-acceleration“ which evidently *equals* the cause with its effect. And from this the reader might learn about the intrinsic materialist philosophy of modern science. Cohen, however, by avoiding any discussion of the philosophical meaning of Newton’s concepts, and by relativising inconsistencies between Newton’s principles and those of physics, obscures and varnishes the fundamental difference between Newton’s thought and the materialistic result of the corruption which Newton’s theory suffered during the past 300 centuries.

5. On p. 34, Cohen confusingly explains the dependence of the relation „weight to mass“ (which is a function of the distance from the centre of the earth only) through „the physical condition of bodies immersed in a fluid or medium,“ stating that e.g. „the buoyancy of air“ might „diminish the weight“ of the body and thus also the said relation. The whole paragraph is physically mistaken and should be cancelled therefore, since, in the case of a body immersed in air, the gravity, and the body’s weight, is not diminished, nor is the relation „weight to mass“ (or the acceleration of free fall) diminished, rather *there acts a counterforce* of different origin named „buoyancy“ which is able to partly or totally *balance* the gravity of the body. Consider that, scientifically spoken and mathematically expressed, it is not the same thing to call a person drifting in a swimming-pool *really weightless* according to an assumption ‘*weight = zero, thence weight to mass = zero’*, or to understand that the swimming person’s still existing weight *G* (of constant magnitude at a constant distance from the centre of the earth) is only *balanced* by a counterforce ‘*minus G’* resulting from the buoyancy of the water. The case, by the way, shows again that Cohen is not able to correctly distinguish between conditions of force and motion *in free space* (as presented in *Principia,* book 1), and conditions of force and motion *in a resisting medium* (*Principia*, book 2) respectively against outside resistances, which has been already criticized in my letter of Aug. 8 with reference to p. 99 of Cohen’s „Guide“ (the wrong „truck“ and the „projectile“ examples for a „vis inertiae“).

6. On p. 39, line 3 and 4 from below, Cohen states that the *passive* forms of Latin verbs relating to orbital motion („gyrare“, „revolvere“, „movere“) which Newton used, may be translated into the *active* English equivalents without altering the sense. The statement is not true, since it makes a decisive difference if one says that a body *„moves“* (as if it were a property of bodies to move), or that a body *„is moved“* (by some external cause, or force, that acts upon the body). The difference concerns exactly the case between a materialist worldview, where phenomena are explained by properties of matter, and Newton’s philosophy of nature, where phenomena are explained as effects of active immaterial forces, or causes. - This, again, is an instance for Cohen’s ubiquitous tendency to ignore the philosophical meaning of Newton’s teaching, and to obscure its difference from the materialist foundations of modern science.

7. On p. 56 line 6, Cohen introduces „*instantaneous* forces of impact and *continuous* forces of pressure“ (my italics) that should appear in Newton’s definitions and laws. Contrary to this statement of Cohen, neither the term „instantaneous“ nor the term „continuous“ appears at the quoted place. Moreover, the idea that a force could generate a proportional motion *instantaneously*, that is *without consuming time*, is physical nonsense, since it implies that a body at rest, when forced to move, could acquire a certain quantity of velocity *without any elapse of time*. And this is equivalent to stating that the body at the same time could stay at different places in space. Of course this idea, which already Galileo rejected, in his "Dialogo" of 1632, appears nowhere in Newton’s teaching. But one should be well aware that it is present in the foundation of classical mechanics, to make evident another fundamental

difference of that theory to Newton's. Moreover, the reader's attention should here be called to the fact that it was one of the main achievements of modern physics (special relativity) to eliminate the wrong „classical" concept of instantaneous generation of motion in favor of a more realist view.

8. On p. 57 in footnote 47, Cohen quotes from Richard S. Westfall that „Newton adhered to *'a dynamic mechanical philosophy in contrast to a kinetic'*". Besides the repeated misuse of the Leibnizian term „dynamics" as a designation of Newton's philosophy, there appears a strange confusion of ideas concerning the theory of motion, since „kinetics" as well as „dynamics" without any „contrast" refers to the theory of force and motion. What Westfall should have said, and what Cohen should have clarified, is the fact that Newton added to the (Aristotelian-Cartesian) theory of only matter and motion the concept of force as 'cause of motion', thus dramatically altering the theory of motion from a deterministic into a *causal* one. An informative „Guide" to the *Principia* might have given a hint to the fact that classical mechanics after Newton fell back to the concept of determinism, while somewhat later modern physics (special relativity) in some way restored the concept of causality, i.e. of a *generation* of motion in space and time.

9. On p. 58, 59 and 60, Cohen comments on the scientific work of Betty Jo Dobbs who has shown that „all issues of passivity and activity, of mechanical and nonmechanical forces, were emeshed for Newton in a philosophical/religious complex" (p. 58 paragraph 4). The aim of Cohen's comments is to make the reader believe that he may and should leave all that stuff aside, and read the *Principia* „as a work in physics", because (as Cohen alleges on p. 59) „the mature Newton put forward and maintained a sharp distinction between 'experimental philosophy' and areas of philosophy that reached significantly beyond that which could be established empirically". To counteract this unproven statement it is enough to quote from the *Scholium Generale* which Newton added to the *Principia's* second edition (1713), saying that „to treat of God from phenomena is certainly a part of experimental philosophy" (cf. Cohen-Whitman p. 943, footnote nn).

Cohen, however, again without any proof, knows better, pretending (p. 60 line 2) that „this was clearly a theologico-philosophical appendix and was never considered to be an integral part of the *Principia*." From this authoritarian statement it is very evident that Cohen aims at garbling the philosophy of nature of Isaac Newton, to make the *Principia* appear as an a empirical-positivistic (say materialist-atheistic) science of today. At this point, Cohen must be accused of not only meanly running down the work of the late Betty Dobbs and others, but also of deliberately ignoring Newton's philosophical legacy in favor of the materialist-atheistic work in merely 'physics' to which it was reduced and corrupted during 300 years of misinterpretation. How weak Cohen's position here is can be seen on p. 60 where he, from the long time it took scholars (boldly including himself!) to „reveal" the „holistic [?] view of Newton's thought", tacitly infers the irrelevance of these findings, which - as he pretends - „would never have been discovered by merely reading Newton's text." Here one meets with an unphilosophical scientist's displeasure at new discoveries which he, though he throughout his long life worked on the *Principia*, was never able to find, because he never asked himself the philosophical question as to which end Newton wrote this book on natural philosophy. Had he done so, Newton would have answered him, in his „preface" of 1686, where he clearly exhibits to aim not at mechanical arts, but on philosophy („Nos autem non Artibus sed Philosophiae consulentes"), and at the end of the *Scholium* following def. 8, where he reveals his very goal through the words: „Hunc enim in finem tractatum sequentem composui". I shall expand this point immediately.

10. On p. 60, Cohen tells us that „one of the most extraordinary things about the *Principia* is that Newton could so thoroughly eliminate the traces of his concern for topics which today we would consider nonscientific."

Let me investigate this statement with respect to the three Newtonian topics „absolute motion", „absolute space", and „absolute time" which indeed are considered today non-scientific 'metaphysical' concepts. All three of them are extensively treated as basic for Newton's teaching in the said *Scholium* following def. 8, a *Scholium* that was part of the *Principia* from its very beginning in 1687. Is that the way to „eliminate traces"? Let me also consider Cohen's statement with respect to the again 'metaphysical' idea of God. As we know, Newton introduced into the second edition of the *Principia* the *Scholium Generale* which makes this idea a natural part of experimental (!) philosophy. Is that the way to „eliminate traces"?

And once more back to „absolute motion". At the end of the said *Scholium* following def. 8, Newton explains how to determine the difference between absolute and relative motion, and finally writes: „But in what follows, a fuller explanation will be given of how to determine true [i.e. absolute] motions from their causes, effects, and apparent differences, and, conversely, of how to determine from motions, whether true or apparent, their causes and effects. *For this was the purpose for which I composed the following treatise."* (Cohen-Whitman p. 415; my italics). That is in Latin: *Hunc enim in finem tractatum sequentem composui*. Newton, therefore, explicitly composed the *Principia* in order to demonstrate the possibility of scientifically treating *absolute*, i.e. *real* motion, and of distinguishing it from *only relative or apparent* motion. It was the aim to mathematically demonstrate the reality and truth of something which, for nowadays relativist physics, is a nonscientific metaphysical phantasm, even though it is, speaking of the true motion of the earth, a philosophical essential of the Copernican cosmology.

The work of Newton thus can be correctly understood as meant to provide a proof for the Copernican and Galilean *theory of the real motion of the earth*, and thus as a work that aims at demonstrating the very eternal *truth* of the Copernican cosmology, and the possibility of a really *true* scientific theory of motion, as a foundation of a *true philosophy*. This *aim at truth* is the special merits of the *Principia*, and distinguishes it from simple physics or 'dynamics' as presented e.g. by the relativist G.W. Leibniz, e.g. in the famous Leibniz-Clarke correspondence of 1715/1716. Significantly this correspondence which on Samuel Clarke's part was mostly written by Newton himself, and which is a store-house for those who want to understand Newton's thought, is not at all mentioned in Cohen's „Guide", nor is the name Samuel Clarke. Generally spoken, Newton's aim at an eternally *true* theory of motion again distinguishes his teaching significantly from 'classical mechanics', and from modern physics as well, since both are based on the principle of equality of all systems of reference wherein *the principle of relativity of motion*, as it was held by e.g. Leibniz against Newton, finds its modern expression. The whole of Cohen's „Guide" has not one single word for this most important concern.

11. On p. 60-64, Cohen expands his idea that Newton adopted a mode of presentation that enabled him to deal with forces „without having to consider any larger questions of a philosophical kind" (p. 61). This mode Cohen calls „the Newtonian style", alleging that he thus was „merely systematizing Newton's own procedure in the *Principi*a and restating what Newton himself has said." However, the examples that Cohen quotes for his allegation only

show that *it is he* who maintains a „sharp distinction between experimental philosophy and the rest of philosophy“ (p. 63) which never was a distinction which Newton had taught. As a matter of fact, Newton, whenever he claims to erect natural philosophy not on philosophical hypotheses, but on phenomena, never argues *against philosophy*, but *against hypotheses* that have no empirical basis, since he aims at a new ‘experimental philosophy’ to proceed from sensual experience to reasonable knowledge of the truths of nature (so says also the „Guide“ on p. 63 paragraph 1).

12. On p. 63, paragraph three, Cohen incidentally makes the reader believe that Newton dealt with „forces acting at a distance“. As a matter of fact, this concept is part of classical mechanics, as a consequence of the materialist tendency to attribute force to bodies as their properties, especially to attribute the ‘centripetal force’ of gravity to the central body as its ‘active gravitational mass’. Consequently, the central body should be able to somehow instantaneously ‘grasp’ to any distant body in order to ‘attract’ it. Newton’s authentic theory, however, understood the centripetal force not as some central agent which (instantaneously!) acted „at a distance“ on bodies distant from the central body, rather he, as can be read in def. 8, conceived it „as a certain efficacy diffused from the centre *through each of the surrounding places in order to move the bodies that are in those places“* (Cohen-Whitman p. 407, my italics). According to this statement, the centripetal force acts *not at a distance*, rather *at the very place where the attracted body is.* And this interpretation corresponds very well with Newton’s statement that „a [central] body cannot act where it is not“, as quoted in Cohen’s „Guide“ on p. 62, line 5. Consequently, one has to understand the accelerative centripetal force (Newton’s def. 7) that is „diffused from the center“ (def. 8, Cohen-Whitman p. 407) in the sense of an immaterial *field of force,* as it was already understood decades ago by Max Jammer, whose findings Cohen „disdainfully and cavalierly“ ignores (to make use of some terms which Cohen attributes to Shandrasekhar’s ignorance of e.g. Cohen's work, on p. 295).

13. On p. 70, line 1-2, Cohen tells us that Newton, only when „he came to understand the way in which forces produce changes in motion, ... invented the name ‘centripetal force’“ [for these forces]. The reader, however, to his confusion finds in the *Principia*, def. 4, and in the second law of motion that forces which produce changes in motion are called not „centripetal“, but „impressed motive forces“, and are *distinguished from* the centripetal force which, according to def. 4, is *only a source of* impressed forces. Here Cohen, though he is well aware that Newton knew various kinds of forces, relapses into the general false belief that Newton had only one concept of force, i.e. the accelerative force of classical mechanics.

14. On p. 88, footnote 6, Cohen misleadingly attributes to Newton the opinion that „the weight of any object on the earth will vary from one latitude to another“, so as if the weight of bodies, insofar as it is identical with the centripetal force, should necessarily differ when measured at different latitudes. See also p. 104, line 7 from below, where Cohen without any further explanation states that the acceleration of free fall „differs from one latitude on earth to another“. From Newton, *Principia*, def. 8, however, we know that the weight of a body „near the surface of the earth, where the accelerative gravity, or the force that produces gravity, is the same in all bodies universally, the motive gravity, or weight, is as the body, but in an ascent to regions where the accelerative gravity becomes less, the weight will decrease proportionately and will always be as the body and the accelerative gravity jointly“ (Cohen-Whitman p. 408). So we understand that the weight of a body depends on its distance from the centre of gravity alone, and from latitudes only if their distances from the centre of gravity should vary. Consequently, the misleading footnote 6 on p. 88 should be cancelled. In fact, the idea of weight varying at different latitudes refers to conditions of rotation of the earth to

produce a *centrifugal counterforce* that partly makes up for the full original centripetal force, i.e. the weight, conditions which are not a subject of the theory of motion of book 1, but of book 3 (see cf. also above, nr. 5).

15. On p. 89, line 1, Cohen quotes Einstein's comment on Newton's experiments concerning the proportionality of mass and weight. According to Einstein, Newton believed that in mechanics „the only possible proof is by experiment", and Cohen seemingly agrees. - This idea contrasts remarkably to the fact that the *Principia* presents not physical questions and their experimental proofs, but heaps of abstract *propositions* and their *geometric* proofs. It is true that physics, since it has abandoned the geometric method, relies on the probative force (if there is any) of experimental evidence only. But not so did Newton. The role of experiment in his mechanics was to establish *that* experience, and *that* knowledge of the phenomena, from which natural philosophy always had to begin (cf. Cohen-Whitman p. 63, line 1). But *demonstration* and *proof* of propositions were *always and only the part of geometry.* There could hardly occur a greater difference between *physics* and Newton's *natural philosophy* than in this difference of methods. Newton's method aims at producing by geometric demonstrations real knowledge and *real truths*. Physics aims at experimentally confirming 'theories' (which Newton would have called 'hypotheses') to provide *preliminary* truths that hold only until they become overthrown in the future by other theories fitting better to experiment. Thus one could say in general that Newton's philosophy by means of geometry aims at truth, while physics by means of arithmetic-algebraic description of experimentally confirmed methods aims at successful applications in technology. And this methodical difference follows certainly again from different most elementary philosophical principles.

16. On p. 89-95, Cohen discusses Newton's definition of 'mass', calling it on p. 89 „one of those qualities [of matter] which ... cannot be intended and remitted", and calling it „a primary property" [of matter] on p. 93. - All these statements have no basis in the *Principia.* Quite on the contrary, Newton explicitly defines 'mass' *not as a quality or property*, but *as a quantity* of matter in def. 1. This definition inevitably leads the unprejudiced reader to an understanding of Newton's concept of 'mass' as based on Newton's knowledge of the 'atomistic' structure of matter - atomistic in the sense of the Ancients, which is to say that all material bodies consist of undividable elementary particles of equal shape. Consequently, the total number of such particles that is contained in a macroscopic body is the 'quantity of matter' in that body. Having understood this concept, no reader of the *Principia* could ever believe in a „circularity of Newton's definition of mass", as Cohen discusses it on p. 89-92.

It is true that Cohen on p. 95, referring to Henry Crew (1935), generously admits that „Newton could very well have had in mind that mass is a measure of the number of fundamental corpuscles or particles or matter in a given sample". However, this interpretation is not only a *possible* one, as Cohen pretends, rather it follows *inevitably* from various places in the *Principia* and other writings of Newton.

As Cohen correctly states here that this interpretation „would eliminate the seeming circularity in Newton's definition of mass", it is surprising that he on p. 89 gives space to the erroneous view of the „eminent philosopher of science Ernst Mach" who, only from fiercely maintaining (in the footsteps of Leibniz and Kant!) a wrong anti-atomistic prejudice as to the 'continuous' structure of matter, by ignoring Newton's atomistic concept could take it into his head to criticize Newton's definition as being 'circular'. Cohen's reverence for Ernst Mach and his prejudiced wrong interpretation is the more surprising as Cohen thus does this well-known

militant Anti-Newtonian the honor to be the only German interpreter of Newton named in his „Guide“.

17. On p. 92, after having told the reader about mass, length, and time to have been „the fundamental units of classical dynamics“, „associated with dimensional analysis (based on M, L, T)“, Cohen pretends that „in the *Principia*, Newton is generally not concerned with units or with dimensionality, because the *Principia* sets forth a dimensionless physics.“

I have already shown in my letter of Aug. 8 how inconsistent and wrong this statement is. Let me, however, refer to another inconsistency in Cohen's presentation. On p.. 92, paragraph three, Cohen states that „Newton's primary quantities *are not* the mass, length, and time of post-Newtonian classical physics.“ But immediately afterwards he writes that „Newton's primary quantities *are* thus mass, momentum, force, space, and time“. Cohen leaves it to the confused reader to find out in what respect Newton's primary quantities „mass, length, and time“ might perhaps differ from „mass, length, and time of post-Newtonian classical physics.“

18. On p. 96-101, Cohen's concern is the „vis insita“, or „vis inertiae“, of *Principia*, def. 3. I have already shown the main inconsistencies and errors of that presentation in my letter of Aug. 8. It should once again be stressed that Cohen, as he on p. 97, line 20 asserts that - according to Newton - „this force is exerted by a body in order to resist a change in state“, misinterprets Newton's words to read: „Exercet vero corpus hanc vim solummodo in mutatione status sui“. This means that the body „exerts“ this force *in changing its state;* not does it mean that the body by this force in any way *„resists“ against* changing its state. One must see that a body, were it not endowed with this innate force, would not 'unresistedly' respond to an outside force acting on it, rather this body would *not at all react* on an acting outside force, i.e. it would *not at all* change its state of motion or rest according to that acting force, which is to say that no interaction between the outside force and the body would take place. Accordingly, the „vis inertiae“ is exactly that means which only makes possible, and arranges any matter-force-interaction. It is due to ignoring the true meaning of this concept of innate force that Cohen, on p. 99, presents two „examples to illustrate what Newton means“ which both are evidently mistaken, *as they both belong to motions against outside resistance* (which is the topic not of book 1 to which Cohen's definition of „vis inertiae“ belongs, but of book 2 of the *Principia;* see above nr. 5, and my letter of Aug. 8).

19. On p. 101, line 18, Cohen writes that „with the elimination of the word 'force', Newton's concept [of the vis inertiae, the inertial force] became the standard of classical physics.“ In my letter of Aug. 8, I have already stressed the point that Newton's concept „with the elimination of the word force“ *is no longer Newton's concept*, since it is the very 'concept of force' alone that makes Newton's teaching a genuine *Newtonian* theory, in contrast to the Cartesian materialistic concept of matter and motion only. See also nr. 21,23,25, and 30 (3) below.

20. On p. 102, line 9, Cohen, referring to Newton's def. 4, pretends „percussion“ (or impetus)“ to be a „source“ of „impressed force“. Newton's Latin says: “Est autem vis impressa diversarum originum, ut ex ictu, ex pressione, ex vi centripeta.“ There is no reference to the term „impetus“. If Cohen wants the reader to believe that the term „impetus“ which Newton makes use of e.g. in def. 3, should mean the same as „percussion“, he ignores everything which has been written during the part 30 years on this subject. In fact, as it can be seen in def. 3, the term „impetus“ means a *force*, that is a *cause* of (duration of) motion, which force may be *transformed* into an impressed force that is „exerted“ during the process of change of motion only, but is not itself a „source“ of an impressed force. Cohen, however,

introduces this strange term „impetus“ without any guiding explanation for the common reader, presumably because it does not fit in with his aim at presenting an unphilosophical interpretation of Newton’s philosophy of nature.

21. On p. 102, line 7-6 from below, Cohen makes Newton say „that his assignment in the *Principia* is the task of mathematicians“ only. The phrase tacitly refers to Newton’s def. 5, where he explicitly says that it is a task for mathematicians to find the measure of a certain force that makes a body move in a certain curved line, and inversely. Nowhere does Newton express a view as if „his assignment in the *Principia*“ *in general* was only a mathematical one, as Cohen suggests. Actually, there is a great many of ‘physical’ and philosophical, even of theological work to be explicitly and implicitly found in Newton’s *opus magnum* - but not, of course, in Cohen’s „Guide“ to this book, which „Guide“, explicitly and implicitly, aims at suppressing all that work in favor of a corrupted positivistic-materialistic presentation of Newton’s teaching.

22. On p. 103, line 11-12, Cohen alleges that Newton, in def. 7, „is subsuming the second law of motion for continuous forces.“ - In fact the second law refers only to the „vis impressa“ which is *not* a continuous, rather an ‘impulsive’ force, to confuse even Cohen, see e.g. the „Guide“, p. 111 below, p. 112 paragraph two, p. 113 paragraph 1.

23. On p. 103, line 15, Cohen pretends that it is the accelerating measure of centripetal force (defined in def. 7) „which Newton generally has in mind in book 1, up to sec. 11, whenever he refers to force.“

Quite the contrary, Newton always distinguishes clearly between „impressed force“ (def. 4), and „centripetal force“ (def. 5-8), as it for instance can be seen in his „Corollaries“ to follow the laws of motion (cf. Corol. 2, 4, 6). Moreover, it is true that the *Principia* shows a „primacy of impulsive [impressed] forces“, as Cohen is well aware (p. 113 line 1). Accordingly Newton, in the *Scholium* to follow def. 8, explicitly states that „true motion is neither generated nor changed *except by forces impressed upon the body“* (Cohen-Whitman p. 412, line 10-11; my italics); and the same thing follows immediately from Newton’s first law of motion to read that a body changes its state of rest or uniform motion if, and *only* if there act „*forces impressed* to change its state“ (Cohen-Whitman p. 416; my italics). Thus one can see that the very „*centripetal force*“ is that purely mathematical concept, whilst „impressed force“ means a real, generating agent, for Newton an *active principle of nature* to generate, and to change in proportion the motions of bodies (according to the second law of motion, see nr. 24, nr. 29). Consequently, even the accelerated motion of free fall according to the centripetal force of gravity is ‘physically’ to be understood as a succession of not accelerations but „equal velocities“ (!) generated in equal particles of time by „impressed forces“ that *originate* (as Newton says it in def. 4) from uniform gravity; says Newton: „When a body falls, uniform gravity, by acting equally in individual equal particles of time, *impresses equal forces upon that body and generates equal velocities“* (Cohen-Whitman p. 424 line 6-8; my italics).

24. On p. 103, line 21-22, Cohen states that „quantity of motion, or momentum, is proportional to mass“. From this idea he infers that „the motive quantity of force must be proportional to mass“, and he feels „the anticipation of the second law of motion“ here.

This statement shows that Cohen neither understands the physical sense of „quantity of motion“, nor that of ‘proportionality’, and consequently also misses the point of the second law to read „Mutationem motus proportionalem esse vi motrici impressae“. To say it in detail:

1) The „quantity of motion“, according to Newton’s def. 2, which is equal to the modern term ‘momentum’ p (so says Cohen, on p. 95 line 3 from below), is given through the product of mass and velocity, p = *m*V. Consequently, a general *proportionality* of momentum p and mass *m*, p/*m* = V, as alleged by Cohen, would require V to be generally *a constant* which it is not. From whence it follows that Cohen does not understand the sense of „quantity of motion“ nor that of ‘proportionality’.

2) Newton’s second law states a proportionality of „mutatio motus“, i.e. „change of motion“, and „impressed motive force“. Given that „change of motion“ may be symbolized by Δ(*m*V), and „impressed motive force“ by F, we obtain F ∝ Δ(*m*V), i.e. F = Δ(*m*V) × *constant of proportionality*, to represent the contents of the second law; but never shall we obtain that *proportionality of motive force and mass*, F ∝ *m*, which Cohen here wants to see in the second law - contradicting, by the way, his own (likewise wrong) statement of the second law in his „Newtonian Revolution“ (p. 177, 193) to contain a proportionality of force and acceleration, F ∝ A, with mass *m* serving as constant of proportionality; and also contradicting his „Newton’s Second Law and the Concept of Force in the Principia“, a treatise of 1970, wherein he votes for „Force = k × (ΔmV)“ with a mere number „k“ serving as constant of proportionality. From whence it follows that Cohen, presenting three different versions of the second law, all of them mistaken, completely misses the point of this most elementary mathematical principle of natural philosophy to be found in Newton’s *Principia*: the *principle of generating cause and generated proportional effect.*

25. On p. 103, line 33, Cohen, referring tacitly to def. 8, states that „Newton insists that his concept of force ‘is purely mathematical’, that he is ‘not now considering the physical causes and sites of forces’“. In the context of this paragraph which begins „Throughout the *Principia“* Cohen compels the reader to erroneously believe that Newton *generally* considers ‘force’ as a purely mathematical concept without a physical meaning. - Newton, however, at the quoted place explicitly refers to his concept of *centripetal* force only. How far Cohen’s conclusion would detract the reader from the true meaning of Newton’s theory of force as an (active or passive) *agent*, and *cause* of motion, can clearly be seen (not only) in Newton’s phrase on the generation of „true motion“, as it is quoted above, in nr. 23.

26. On p. 104, paragraph two, Cohen alleges that „Newton is dealing with relative rather than absolute quantities“, and from this he infers that, whenever Newton computes the weight of e.g. equal bodies, this weight „is equivalent to the weight per unit mass.“ This statement is mistaken in two respects.

1) As we have seen above (nr. 15), ‘mass’ is for Newton the absolute total number of elementary material particles in a body. Consequently, whenever he deals with mass, this means always a number which is an *absolute* quantity. Cohen was right if Newton would refer masses to a certain conventionally defined ‘mass unit’, as e.g. to a prototype body of approx. 2205 lb. kept at Sèvres near Paris called ‘kilogram’. But this is not what Newton does, and Cohen is right in stating on p. 92, line 12 that Newton „does not establish a [conventional] unit of mass and then compute individual masses“ relative to it.

The case is the same with respect to Newton’s measurement of ‘spaces’ and of ‘times’. Newton does not refer to any conventional scales of ‘feet’, ‘fathoms’ or ‘miles’, of ‘seconds’ or ‘hours’, rather *he measures spaces in units of absolute space, and times in units of absolute time*, as he explains it in the *Scholium* that follows def. 8. He does not use a pocket rule or a

tape measure, he does not use a watch, rather he makes use of *absolute space* as an infinitely extended scale of lined-up „parts of space“ (Cohen-Whitman p. 410, line 12 from below) to measure a quantity of „relative space“ as „any movable measure or dimension of this absolute space“ (Cohen-Whitman p. 409, line 1), and he makes use of *absolute time* as an infinitely extended scale of lined-up „parts of time“ (Cohen-Whitman p. 410, line 11 from below) to measure a quantity of „relative time“ as a „sensible and external measure of duration“ which duration is absolute time itself (cf. Cohen-Whitman p. 408, line 6-5 from below). As a consequence, the quantities of ‘spaces’ and of ‘times’ which he compares throughout the *Principia* are always quantities, or ‘dimensions’ made of *absolute* space and *absolute* time, and ‘relative’ only in so far as they are measured *in relation to the infinite scales of absolute space and absolute time.* Says Newton, e.g.: „Absolute and relative space are the same in species“ (Cohen-Whitman p. 409 line 6).

Generally spoken, contrary to Cohen’s allegation Newton is dealing with absolute rather than relative quantities *whenever* he deals with quantities of masses, times, and spaces (and motions!) in book 1.

2) Consequently, whenever Newton is dealing with e.g. „weight“, this weight to spite Cohen (p. 104, line 16) is *not* “equivalent to the weight per unit mass“, rather it is that *absolute* „motive quantity of centripetal force“ which Newton defines in def. 8 through the product of „accelerative force and [absolute] quantity of matter jointly“ (Cohen-Whitman p. 407, line 3 from below), calling it clearly by the name of „weight“ (Cohen-Whitman p. 407, line 15).

27. On p. 104, Cohen wants the reader to understand his view of Newton’s measurement of „weight“ by the help of Newton’s second law which he presents here in the form $F = mA$, ‘force F equals mass *m* times acceleration A’, the most elementary principle of classical mechanics.

1) Cohen knows very well that this formula does *not* represent Newton’s second law, as it can be seen on p. 111-117 of his „Guide“. However, putting aside this objection for the moment, let us see what use Cohen makes of this law. He tells us that the accelerative measure of the force is $F/m = A$, but he does not understand that this measure is exactly a proportional equivalent not of „impressed force“ as it is the concern of the second law, but of the „accelerative quantity of *centripetal* force“ defined by Newton in def. 7 as ‘centripetal force *per particle of matter in a body’*. Consequently, Cohen moreover wants the reader to believe that „for constant masses *m*, or for a unit mass, the measure of force is [not mA, but only] A“ which assertion is evidently wrong: In fact, the measure of ‘force’ (in classical mechanics) is never only A, but always mA, whether or not one considers a „constant mass“ of an extended gross body, or a „unit mass“ only. This can be seen from the dimensions [mA] of „force“: Even if, in the case of a „unit mass“, *m* should equal „one“ (i.e. $m = 1$), this *m* nevertheless keeps its dimension [*m*], so that the force $F = 1 \times A$ keeps its dimensions [mA] as well, and may never be put equal to the acceleration A [L/T^2] *alone*, as Cohen suggests it. Whenever Newton, by the way, seems to follow this suggestion, in fact he makes us not of an *identiy* $F = A$, but of *the proportionality* $F \propto A$ between force F and acceleration A (cf. def. 7) which is allowed *mathematically*, supposed the quantity *m* is one and the same in all the cases considered.

2) As Cohen knows about the difference between the $F = mA$ of classical mechanics and Newton’s second law (cf. the „Guide“ p. 92, p. 111-117, and p.296 line 1-4 where he harshly criticizes the late S. Chandrasekhar for incorrectly equating Newton’s „change in motion“

with „mass × acceleration“, even though this alleged „Newtonian“ equation is the starting point of all textbooks of classical physics all over the world), the reader must once more be surprised to find him harmlessly making use of F = *m*A on p. 104. It should always be seen clearly, and it should be stated explicitly that there lies an abyss between Newton's second law and the „F = *m*A“ of classical mechanics , a fact that is well-known among historians of science, though only few of them (not Cohen) have already understood its importance as a key to the authentic teaching of Newton. And the abyss, again, concerns the question if 'force' is nothing but a special mode of motion, say accelerative motion, with which it is to be identified - or if it is something different from material motion, something that *causes* material motion as its *effect*, which something *is not equal, but proportional* to its effect, as the second law of motion states it to reveal its character as a real *law of nature*, which a simple *equality* of force and motion is not. The abyss, then, concerns to distinguish between a materialistic philosophy present in the F = *m*A -interpretation of the second law, and a philosophy that accepts 'force' as an immaterial agent which is *proportional* to its effect, F ∝ *m*A, or F ∝ Δ(*m*V) according to the correct „impulsive“ form of the second law. One has no choice between these interpretations but to adopt a definite philosophical position, pro or contra materialism.

28. On p. 104 line 7 from below, Cohen once again states that the acceleration of free fall „differs from one latitude on the earth to another“. I have already refuted this erroneous statement on centripetal force (erroneous in the context of Newton's book I) in nr. 14 above.

29. On p. 105, Cohen again wants the reader to believe „that throughout most of the *Principia*, especially the first sections of book 1, Newton makes use of the accelerative measure“ (of force), and, „generally spoken, whenever Newton introduces 'force' without any qualification, he intends accelerative measure.“ Contrary to this allegation, it can easily be seen by reading the *Principia* that Newton nearly always when he wants to speak of a centripetal force, carefully adds the word „centripetal“ in order to make clear what he is speaking of. And to spite Cohen this is very evident especially in the first sections of book 1: Sect. II „De inventione *virium centripetarum*“, Sect. III Prop. XI „Revolvatur corpus in Elipsi: requiritur lex *vis centripetae* tendentis ad umbilicum ellipseos“, etc. etc.

I suppose that Cohen here is misguided by the general belief (of physicists who identify Newton's teaching with classical mechanics) that 'force' must *always* mean something that deals with 'acceleration'. Quite the contrary, Newton in the first place means always force as something that generates 'motion', i.e. 'mass times velocity' (def. 2), or 'change of motion', as it is expressed in the first and the second law. Most curiously, Cohen is well aware of this fact, as it can be seen e.g. on p. 113 line 1 („The primacy of impulsive forces“). Sometimes the reader, due to the various inconsistencies, must be suspicious that in composing the „Guide“ at least two persons with different views of the *Principia* have put their shoulders to the wheel.

30. On p. 105, line 8 from below, we read that in the *Principia,* „the level of physical discourse undergoes a major shift in sec. 11, going from 'accelerative' to 'motive' forces and thereby inaugurating the science of dynamics [!]. From a strict point of view, the subject of book 1, secs. 1-10, is largely kinematics despite the use of the term 'force'“.

The whole phrase has nothing to do with the *Principia,* and should be cancelled therefore.

1) Newton, in his introduction to sec. 11, explains clearly what sort of a „shift“ is going on here: „Up to this point, I have been setting forth the motion of bodies *attracted toward an immovable center*...I now go on to set forth the motion of bodies that *attract one another*, considering central forces as attractions...“ (Cohen-Whitman p. 561, my italics). - Not a single word of Newton’s fits here with Cohen’s allegations.

2) To say it once again: The „science of *dynamics*“ is a science of motion in the Aristotelian-Leibnizian style, i.e. a science of *motion against resistance*, as from friction etc., resting on concepts of „kinetic energy“ and „work“ that were developed by Leibniz and his school. Already the elementary concept of ‘force’ of Leibniz, as is well-known, was conceived and computed according to the idea of motion *against the resistance of gravity* (i.e. lifting a given weight through a given distance), and consistently led him to the measure mV^2 of ‘energy’. Newton, on the contrary, in book 1 deals with *absolutely unresisted motion*, i.e. with motion in absolutely empty space void of any resistance whatsoever. The matter concerns Leibniz’s Cartesian attitude to emphatically (and erroneously) deny the existence of a void, contrary to all the true teaching and experience of Galileo, Torricelli, Guericke, and Newton, of course.

3) Cohen’s idea to call Newton’s teaching in book 1, sect. 1-10 „kinematics despite the use of the term ‘force’“, shows again how little he cares for Newton’s true and clear words. Since the term „kinematics“ indeed marks a theory of motion that does not consider any generating forces, it is absolutely incomprehensible how Cohen dares say that Newton presents anything like that, were it not for the reason that Cohen presumably adheres to the anti-Newtonian materialistic philosophy which, by attributing any effects of any ‘force’ only to properties of matter itself, in fact *banishes and eliminates* ‘force as an immaterial agent’ from mechanics. Note that this was what Leibniz (not he alone, but also d’Alembert, for instance) strongly advocated against Newton, e.g. in his above quoted correspondence with Samuel Clarke respectively with Newton himself.

31. On p. 106, paragraph 3 we meet again with Cohen’s advice that Newton’s *Principia* should *generally* be read as „a purely mathematical text“. I have already shown above as to which extent this advice leads into error. As Cohen here refers to Derek T. Whiteside’s really famous edition of „Mathematical Papers“ of Newton, the reader should be warned that Whiteside’s extended work on Newton must be understood as a translation of Newton’s geometric writing into the algebraic language of the Leibnizian *calculus*. For this statement I refer to Niccoló Guicciardini, „Reading the *Principia*“, Cambridge 1999, p. 2 paragraph one. It will be shown elsewhere in which direction Whiteside through this translation inevitably alters the true meaning of Newton’s teaching.

32. On p. 106-107, Cohen informs the reader that Newton „believed in absolute space, just as he believed in absolute time“ - and in absolute motion, and in God, might Cohen have added, in order to concretize what he means with Newton’s „belief“. As a matter of fact, interpreters of the *Principia* throughout 300 past years have successfully disseminated the opinion as if Newton’s science of absolute space, absolute time, and absolute motion was just a matter of belief that could be omitted from his theory of motion without doing any damage to science. From these efforts there originated that „classical mechanics“ of the schools which lacks the said concepts, and consequently presents a (Cartesian-Leibnizian, and Aristotelian) non-Newtonian theory of only *relative* motion, in flagrant contradiction to Newton’s aim and purpose, as it can be read e.g. in his *Scholium* following def. 8. By the way, in his arguments on p. 106-107 Cohen reveals that he has no idea of the meaning of the terms „absolute“ and

„relative“ which cannot be understood but with the help of philosophy. Certainly he pays a heavy price here for his repeatedly emphasized abstinence from philosophical considerations.

33. On p. 107-108, Cohen refers to Newton’s „celebrated rotating bucket experiment“. On p. 108 he quotes from a publication of Robert Rynasiewicz who has shown that Newton’s purpose here was „to convince the reader that ‘true motion and rest can be adequately understood only with reference to motionless places, and hence to absolute space as characterized in the scholium’“.

I think that Rynasiewicz has met the point of Newton’s argument. What follows, then? Cohen, to disappoint the reader, is silent on that question. In my view, it follows again that classical mechanics as a theory of *only relative motion*, defined (according to the ‘principle of relativity of motion’) with reference not to motionless absolute space, but to other moveable ‘bodies (or systems) of reference’, must not be confused with Newton’s authentic teaching on *true* motion. Unfortunately, Cohen the „Guide“, in spite of his repeated protestation to the contrary, due to his ignorance of philosophy is relapsing into that very confusion again and again.

This letter refers to chapters 1-4 of Cohen’s „Guide“ (p. 11-108). I shall expand my criticism to chapters 5-11 in an additional letter.

Sincerely yours ...

Ed Dellian D-14169 Berlin, Sept. 7, 2000

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ISAAC NEWTON, THE PRINCIPIA
A new Translation by I. Bernard Cohen and Anne Whitman
preceded by a Guide to Newton's *Principia* by I. Bernard Cohen

Dear Sir/Madam,

resuming the concern of my letter of August 29, I shall now proceed to inform you on errors and misinterpretations in Prof. Cohen's „Guide", continuing the number of items (1 - 33 in the previous letter).

34. On p. 109, 110, 111, Cohen discusses the question why Newton formulated a first and a second law of motion, and what he presumably might have meant especially with the first law. The whole line of reasoning is un-Newtonian and unacceptable in several respects.

1) Un-Newtonian is already the first phrase (on p. 109) to read: „The first law states the law or principle of inertia which Newton had learned from Descartes's *Principia*." - In fact, there is *no reference made* to such a „principle of inertia" in Newton's law which reads: „Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus illud *a viribus impressis* cogitur statum suum mutare" (my italics). The „principle of inertia", to be sure, is not a Newtonian, rather a materialistic Cartesian one, referring to *a property of matter* called 'inertia' which should make a body persevere in its state. Newton, quite the contrary, in accordance with his numerous statements on the absolute inactivity of „inanimate brute matter" (as expressed e.g. in a letter to Bentley), conceives no such property of matter. According to his def. 3, a „*force* of inertia" (Cohen-Whitman, p. 404, def. 3 line 8; my italics) innate in matter should, as a (passive) cause, generate the perseverance of bodies in their states of rest or uniform straight-lined motion. Once again the difference is utterly important for an understanding of Newton's non-materialistic anti-Cartesian theory of force and motion, based on the knowledge of *passive and active forces of nature* that cause bodies *to persevere* in their state (the *vis inertiae* of def. 3), or *to change* their state (the *vis impressa* of def. 4, and of the first and second law). Consequently, it is not - as Cohen pretends - „the first law" that states something on „inertia", rather this subject is the concern of Newton's def. 3 only.

2) What, then, is the aim of the first law? The law, as cited above, just answers the question which state a body would maintain so long as no „impressed force" compels it to change that state. This law is basic for a causal theory of force and true motion, since it makes clear that *all* those always curvilinear or accelerated or decelerated motions of bodies which we can observe in nature, *are always caused by real „forces impressed" on a body from outside*, because *without the action of such outside forces* (already defined by Newton in def. 4), *the body would persevere in its state of rest or of uniform-rectilinear motion.* Newton thus, opposing the Aristotelian-Cartesian materialist theory of motion, *states that a body will never*

change its state 'by itself', say due to an imagined self-regulating property of matter to move by itself, rather by active forces acting from outside on the body only. Its is this „only" (corresponding to the „nisi quatenus" in Newton's Latin) which, in the first law, goes beyond the contents of def. 3 and 4 (which are but 'definitions'), and determines its character *as a real 'law' of nature to permit no exception.*

3) On p. 110 line 1, Cohen speculates why the „laws of nature" stated by Descartes in his „Principia Philosophiae" would not „enable him to infer forces from motions". Cohen's reasoning is mistaken because he, shunning philosophical considerations, does not realize the most elementary difference between Descartes and Newton concerning the reality of immaterial 'forces of nature'. If Descartes is not able „to infer forces from motions" this does not follow from his „laws of nature", as Cohen puts it, but only from the fact that he, teaching a materialist theory of matter and motion only, does not conceive any 'forces of nature' in the sense of Newton.

4) Still on p. 110 paragraph one, Cohen once more speculates about Newton's second law, raising the question „why there are both a first and a second law", because somebody might think that the first law, as it refers to „impressed forces" that change the states of bodies, „seems to be an consequence of the second law" which states that these impressed forces are always proportional to the change they have generated in a body's state of motion. - The raised question shows that the questioner is not able to correctly read Newton's laws due to lack of philosophical education. Since he does not know the difference between a 'law' and a 'definition', first he misses the above explained point of the first law as a *'law' unlike the 'definitions' 3 and 4.* And consequently, he secondly misses also the point of the difference between the first law which only *qualitatively* refers to impressed forces that always change the state of motion of bodies, whilst the second law, again *as a law*, goes far beyond the mere *definition* of „impressed force" in def. 4, as it states *quantitatively* that these „impressed forces" are always and without exception *proportional to* the generated change in the state of motion of a body. And this ignorance of the questioner is also due to the fact that he, in order to explain the second law, again makes explicitly use of the equation F = *m*A here, which *ignores the core of the second law, i.e. the Newtonian p r o p o r t i o n a l i t y* of 'cause' F and 'effect' *m*A (F ∝ *m*A), and thus replaces the mathematical evidence of the second law through a Leibnizian *equality* of cause and effect which yields only a *definition* of 'force' at will without any lawlike contents and relations to nature. Consequently, F = *m*A is *not* an adequate rendering of the second law, as has already been shown above, and as Cohen should know very well.

5) Most surprisingly, then, at the end of paragraph one on p. 110, Cohen, after having struggled with the deficiencies of F = *m*A for a while, offers as „a possible clue to Newton's thinking" the unfounded speculation that the first law (though it refers only to „impressed forces" as well as the second) might be „concerned with *a different kind* of force" (my italics). And once again Cohen leaves it to the unguided reader of his „Guide" to consider the validity of a „clue" which only can lead in confusion, as will be seen in the next item. Before Cohen, nobody, including Newton, has ever had an idea of such a mysterious „different kind of force".

6) On p. 110, paragraph two, Cohen raises the question „why Newton believed he needed a first law since the principle of inertia had already been anticipated in def. 3, and also in def. 4." - I have answered this question above, in 1) to 3). Cohen, however, continues to speculate that the first law might have been meant as „a condition for the existence of certain insensible

forces, not otherwise known to us.“ As the „most significant such force“ he then cites „the centripetal force“, our awareness of which, according to Cohen, „is [only] based on the first law and the observed fact that the planets do not follow a uniform rectilinear path.“ - Here one sees clearly Cohen again ignoring Newton’s words, since Newton speaks in both laws of „impressed forces“ to change the states of motion of bodies, as defined in def. 4, not of „centripetal forces“, as defined in def. 5-8. Moreover, the phrase on our „awareness of the centripetal force“ ignores that the world since men live on earth is clearly aware of this force due to the phenomena of fall e.g. of apples from trees. As a matter of fact, the appearance of „impressed forces“ in both the first and the second law, and not of a „centripetal force“ that is only a subject of definitions 5-8, corresponds with Newton’s statement in the *Scholium* following def. 8 that „true motion is neither generated nor changed *except by forces impressed* upon the moving body itself“ (Cohen-Whitman p. 412, line 10-11, my italics). The real „clue“, therefore, how to understand the *Principia* here is to carefully distinguish „impressed forces“, as lawfully acting *real active principles*, from the only mathematical concept of a „centripetal force“ which does not act by itself, rather it is *but a source of impressed active forces*, according to Newton’s def. 4.

7) On p. 111, Cohen, after having alleged on p. 110 that Newton in his „decision to have a separate law 1 and law 2“ followed „the model he found in Huygens’s *Horologium* Oscillatorium“, nevertheless notes „that the conditions of Huygens’s two hypotheses are somewhat different from those of Newton’s first two laws.“ Indeed, as footnote 5 reveals, „in Huygens’s hypothesis 1, the only force under consideration arises *from the resistance of the air*“ (my italics). Here Cohen could very well have informed the reader about that above discussed difference between Newton’s theory of unresisted motion (book 1), and the materialist Aristotelian theory held by Descartes and his followers (Huygens, Leibniz, d’Alembert, Lagrange) which, as it is based on denying a void, deals always and only with motion against resistance. The instance shown by Cohen makes it clear that the whole discussion of a „similarity“ between the concepts of Huygens and Newton is utterly superficial and mistaken, and this again is due to Cohen’s confessed abstinence from philosophical matters.

8) On p. 111 paragraph two, Cohen discusses „three examples of the longtime persistence of linear motion“ which Newton gives after stating the first law. If these examples „on first encounter may seem confusing“, as Cohen says, this has nothing to do with an „analysis of curved motion“, as Cohen wants to find it here. Rather these examples show the reader how Newton is able to discover *from experience* the natural law of a ‘uniform-rectilinear motion ad infinitum’ which (as it was often stressed against Newton by the Cartesians) *never and nowhere in nature can be observed*, because all observable motions due to the actions of impressed forces are either accelerated or decelerated, or follow a curved line, as it is the case with the said „three examples“. Newton here makes use of the means of ‘demonstratio apagogica’, that is *indirect demonstration*: It is *just because* we see all *observable* (accelerated, decelerated, or curvilinear) motions of bodies as effects of forces impressed on the bodies, we may infer with Newton that a body, if and so long as such forces were absent, would persevere in its state as claimed by the first law. The matter concerns the philosophical question if, and in what respect, Newton’s teaching is based on empirical data only, since we have no (direct) empirical knowledge of uniform straight-lined motion. Some interpreters consequently have called this form of motion an unprovable, and ‘metaphysical’ idea. As I see things, Newton shows here (against the philosophy of the empiricists) how we, starting from a strictly empirical basis, can obtain knowledge even about the reality of something which, as it is certainly *not* an empirical phenomenon itself, may well be called ‘metaphysical’ (without

the term's pejorative taste). However, I prefer the term 'transcendent' (which means '*not to be directly perceived by sense'*) for those unobservable things as 'absolute space', 'absolute time', 'absolute motion', and 'uniform straight-lined motion' which all, methodically deduced from phenomena, mean indispensable transcendent principles of Newton's true philosophy of nature.

35. On p. 111, Cohen again deals with the second law, stating loosely that the impressed force of this law „means an impulsive force or impulse." - „Impulsive force" is certainly correct, „impulse" alone is not, due again to the meaning of 'force' as 'cause' in Newton's philosophy which Cohen ignores. Thus, an „impulsive force" means the '*cause* of an impulse', and not the 'impulse' itself which means the *effect* of the causal impressed force.

36. On p. 112 line 5, Cohen informs us that he understands Newton's Latin phrase „simul et semel" in the sense of „all at once (instantaneously)". The Motte-Cajori-translation has „altogether and at once". Now, as the term „instantaneously" carries a very specific physical meaning today which implies 'without consuming any time', and as Newton in a famous letter to Bentley *explicitly rejected* the idea of an instantaneous action at a distance, I doubt this interpretation to be correct, the more since 'instantaneousness' of change of motion would mean that a body miraculously should be able to occupy different places in space at the same time. Consequently, Cohen's misguiding term „instantaneously" should be cancelled.

37. On p. 113, Cohen proposes to „hazard a guess why Newton chose to make impulses [i.e. impulsive, or „impressed" forces] primary, and continually acting [centripetal] forces secondary or derivative." His guess, then, is speculative to the extent that he better had not posed this question. The true answer, however, is easy if one only refers carefully to Newton's words. It is in def. 4 where Newton states that bodies change their motions *only* according to the actions of „impressed forces", i.e. of *impulsive forces*, and not at all according to a „centripetal force" (as defined in def. 5-8) which centripetal force *means only some source* where the only active impressed forces spring from. It is in the *Scholium* to follow def. 8 where Newton states that (to quote it once more) „true motion is neither generated nor changed *except by forces impressed upon the body"* (Cohen-Whitman p. 412, line 10-11, my italics). It is in the first law of motion where Newton states that a body will change its state if, and *only if* there act „[impulsive] *forces impressed* to change its state" (Cohen-Whitman p. 416, my italics); see also nr. 23 above (my letter of Aug. 29). Cohen's unfounded speculation shows that he, against the cited statements of Newton, holds the view of classical mechanics to attribute the name of force only to some matter which, by „action at a distance", should attract material bodies, and make them instantaneously move in an accelerated manner in the direction to its centre. Once again one meets here with the core of the difference between classical mechanics and Newton's true philosophy of force and motion in space and time.

38. On p. 113, line 4-5, Cohen states that Newton's „ultimate goal in the *Principia* was to explore the actions of universal gravity in various aspects." - Newton himself defines his goal differently. See the „Author's Preface to the Reader" (Cohen-Whitman p. 382), *where he stresses his sole concern with philosophy*, the whole difficulty of which „seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces. It is to these ends that the general propositions in books 1 and 2 are directed, while in book 3 our explanation of the system of the world illustrates [!] these propositions." See also the *Scholium* to follow def. 8 (Cohen-Whitman p. 415), where Newton reveals „the purpose for which I composed the following treatise" to have been to show „how to determine true motions from their causes...and conversely ...from motions

...their causes and effects.“ Newton’s „ultimate goal“ to spite Cohen was not restricted to explore „the actions [?] of universal gravity“ (which gravity, as we have seen, according to Newton does not „act“ at all, rather serves as a source of acting „impressed forces“). His ultimate goal was to explore and demonstrate the reality and truth of really acting impressed forces of nature, and of their true actions on material bodies. His aim, then, generally spoken, and *his ultimate goal was true knowledge of nature, or truth itself.*

39. On p. 113, paragraph two, Cohen presents Jacob Hermann to have been „the first person to make a formal statement of the second law in the algorithm of fluxions or the calculus“, and he refers to Hermann’s formula G = M*d*V **:** *d*T. However, Cohen’s allegation is mistaken in two respects.

1) In the equation G = m*d*V **:** *d*T according to Hermann (as Cohen cites) „G signifies weight or gravity applied to a variable mass M.“ It is clear, then, that this equation has nothing to do with Newton’s second law (which explicitly refers to „impressed forces“ as defined in Newton’s def. 4), rather it refers to Newton’s def. 8 of „the motive quantity of centripetal force“ which is „the measure of this force that is proportional to the motion which it generates in a given time“, and which of course means (according to Newton) the body’s „weight“ (Cohen-Whitman p. 407, line 15).

2) Hermann’s equation of 1716 not only makes use of Leibniz’s notation, but also transports the spirit of Leibniz’s „Dynamica“ of 1695, where the mentioned quantity G is defined for the first time as the basic concept of mechanics, called „vis mortua“, the *dead force*, from which, by integration, results the „vis viva“, the *living force* which is present in a body in accelerated motion, as a space integral of the *dead force*. This algorithm is of course again the foundation of classical mechanics, based on the concept of „gravitational force“ or „weight“ alone. And here the historian of science should be able to see, and should tell the reader how classical mechanics differs from Newton’s teaching of motion caused only by „impressed forces“, and how closely this classical mechanics is intertwined with *Leibnizian concepts and philosophy*.

40. On p. 113, in the last paragraph, Cohen refers to Newton’s attribution of the first and second law of motion to Galileo, where Newton is „even alleging [sic] that it was by use of the second law that Galileo had discovered the law of falling bodies.“ Cohen, again, knows better than Newton, stating in an authoritarian manner that „Here Newton showed that he was not aware of how Galileo had presented his discovery. Galileo certainly did not know Newton’s first law. As to the second law, Galileo would not have known the part about change in momentum in the Newtonian sense, since this concept depends on the concept of mass which was invented by Newton and first made public in the *Principia.*“

The whole paragraph is mistaken in several respects, and should be cancelled therefore.

1) Cohen’s idea that Galileo, contrary to Newton’s attribution (see Cohen-Whitman p. 424, line 2-4), had not known the first and the second law of motion presented by Newton in the *Principia*, rests on Cohen’s misinterpretation of both these laws, as I have criticized it above. If one carefully, and without authoritarian prejudice, studies Galileo’s works, one will clearly see that he had full knowledge of both laws of motion in their true sense, even though he does not use *the terms, and also not the algebraic expressions* according to which today physicists understand these laws.

2) As to the concept of „mass“, it is most surprising to read Cohen's allegation that this concept „was invented [sic] by Newton“. In fact, it was perhaps Newton who first used *the term* „mass“ to name the „quantity of matter“, i.e. the number of material particles in a body. *This concept*, however, belongs to the atomism of the Ancients, and was never „invented“ by Newton, nor was it unknown to the convinced atomist Galileo or to other scientists (e.g. Pierre Gassendi) before Newton. Especially that knowledge „about [the role of 'mass' in the] „change in momentum“, to which Cohen refers, had been clearly revealed years before Newton wrote the *Principia*, and therefore Newton in the *Principia* correctly recalls to mind these findings (in the years 1669-1670) of „Sir Christopher Wren, Dr. John Wallis, and Mr. Christiaan Huygens, easily the foremost geometers of the previous generation“ (Cohen-Whitman p. 424).

41. On p. 114, line 7, Cohen again alleges that an „impulsive force“ according to Newton's second law should cause „an instantaneous change in the body's 'quantity of motion', or momentum.“ On p. 115, line 1, he speaks of „instantaneous or impulsive forces.“ - As I have already shown above, the idea of an „instantaneous change“ of motion, i.e. of a change in the motion of a body *without any elapse of time*, is physically nonsense, and is of course nowhere a part of Newton's true theory of motion in space and time. As to the second law, this idea results only from ignoring the explicit *proportionality* which Newton states between the acting force and the produced effect on motion, in favor of an *equation* of force/cause and effect according to Leibniz's [!] *first axiom of mechanics* „causa aequat [!] effectum.“ If in this equation the integer effect of an acting force appears *as if it were produced without any elapse of time*, this is only due to the fact that the constant of proportionality between the producing force and its produced effect is suppressed in favor of their *equality*, implying inevitably their equal appearance at equal times.

42. On p. 115, line 11, Cohen points to „the limiting process introduced in prop. 1 of book 1[which] boldly declares the mathematical character of the *Principia*.“ - Here Cohen, taking into account the various instances for Newton's adherence not to algebra and arithmetic but to geometry, should have explained that Newton nearly always when he stressed the validity and meaning of 'mathematics', meant not arithmetic and algebra, and consequently not the *analysis* as promoted by Descartes and Leibniz, but the *geometry of the Ancients*, as that means which *by synthesis* allows for an exact, i.e. scientific understanding and measurement of force and generated motion as a natural process to really take place in space and time (cf. „Author's Preface to the Reader“, Cohen-Whitman, p. 382, paragraph one).

43. On p. 116, paragraph two, Cohen speaks of „Newton's transition from impulsive (instantaneous) to continually acting forces.“ - Here again, the misuse of the term „instantaneous“ must be criticized, which term nowhere is Newton's, and moreover means something which is physically nonsense. „Impulsive forces“ according to the second law produce certain quantities of motion, or momentum, by generating certain changes in the state of a body. This production of a certain quantity of momentum, say e.g. of a certain velocity of motion in a given quantity of matter or 'mass', will never happen „instantaneously“, but in time, so long as bodies do not miraculously occupy different places in space at the same time.

44. On p. 116, paragraph two, Cohen alleges the forms „(1)$F \propto dV$, (2) $F \propto dV/dt$“ (both of which he erroneously presents as „forms of the second law“) to be equivalent if *dt* is constant. -This statement is mathematically wrong, since form (1) is evidently *not* an equivalent of form (2), no matter if *dt* is a constant or a variable. The instance shows, however, that Cohen does not understand what a proportion really means. Wrong, then, as well is Cohen's subsequent

idea to present the forms „(1a) $F = k_1 d(mV)$, (2a) $F = k_2 d(mV)/dt$“ as supposed *equivalents* of forms (1) and (2), and Cohen is wrong since these forms (1a) and (2a) differ from (1) and (2) in the application of a factor *„m“*.

45. On p. 116-117, Cohen presents his view on „dimensions of proportionality [which] would not have posed a problem for Newton, because he expressed the principles of motion in proportion in a rhetorical style and not in equations. He thus had no need to consider the dimensions of constants.“

To these empty words scholars often resort which simply fail to translate Newton's proportions into synonymous equations.

1) It is absolutely clear in mathematics that constants of proportionality have proper dimensions depending on the dimensions of the proportional quantities themselves, no matter if these constants are given *implicitly* (through the verbal expression of a „proportionality“ of two variables $A \propto B$), or *explicitly* (through an *equation* $A = B \times$ *constant of proportionality*). Consequently, the proportionality of two variables A, B, depends mathematically on the dimensions of the constant, which thus cannot be ignored, even not if the constant is only given *implicitly*, by speaking of proportions „in a rhetorical style.“ See also my letter of Aug.8, 2000.

2) The matter concerns mainly Newton's second law which proportion generations of scholars have rendered into an equation by *simply suppressing* the constant of proportionality between the force „vis motrix impressa“ and its effect „mutatio motus“. One should note that this suppression did not result from a simple *error,* rather it followed from the philosophical idea of Leibniz to *equate* causes and effects which idea is an intrinsic part of the Leibnizian calculus. So, when the *Principia* was rendered into the language of this calculus, the said constant of proportionality got necessarily lost. N. Guicciardini, in his „Reading the Principia“ (Cambridge 1999), shows that with the progress of the analysis on Cartesian and Leibnizian foundations, the theory of proportions got lost, and from there on 'ratios' were „considered as quotients, proportions as equations“ (p. 127). Cf. also Guicciardini on p. 257: „The eighteenth century was characterized by the analytical programme emphasized by the Leibnizian school, while the role attributed to geometry [and to its heart *proportion theory*] by Newton and his followers faded away“. - Cohen, as many others before him, struggles with the problem how to render the second law, proposing different and inconsistent ideas, but ultimately cannot solve it because he is not yet ready to accept that the „second law“ of classical mechanics *differs* from Newton's second law, and that it differs exactly in so far as it suppresses and eliminates a constant of proportionality (the „Newtonian Constant“, as I have baptized it, for the first time in 1985).

46. On p. 117, line 6-7, Cohen rejects the view „that Newtonian time is itself made up of some sort of discrete infinitesimal units.“ If one reads footnote 16 on this page 117, however, one sees that Cohen here does not mean discrete *„infinitesimal“* units of time, but discrete *„finite* time intervals“ in contrast to the *Leibnizian concept* of unextended „infinitesimal“ times *dt* to constitute a time continuum in the way points may constitute a line (if so). Though Cohen, „indebted to [the authority of] D.T. Whiteside“, as he confesses, had to correct a personal error in this matter, he nevertheless (to confuse the reader again) draws on Newton's „De Quadratura“, where Newton „unambiguously referred to ...equal ... very small particles of time.“

It is true, indeed, that Newton, not only in „De Quadratura“, but also on various occasions in the *Principia*, speaks clearly of real finite elementary particles of time, and of space. As a matter of fact, infinitely extended scales of time and of space, subdivided into elementary equal finite particles of space and time, are present already in Galileo's „Discorsi“ of 1638 to serve as a space-time frame of reference for the quantification and measurement of variable 'spaces' and 'times' which quantify the dimensions, i.e. the measure of 'motions' of bodies. Newton expands Galileo's theory on the said foundation. From a philosophical point of view, Galileo and Newton propose conceptions of *real,* and *really 'quantized'* time and space, made up of *discrete* equal elementary particles, which conceptions are wholly different from the space and time *continua* of Aristotelian, Leibnizian and classical mechanics, and differ also - by the way - from the space-time continuum (!) of modern physics. Of course, if one wants only to translate Newton's theory of motion into terms of the Leibnizian calculus, as Whiteside does it, one has to depart from this promising 'quantized space-time frame of reference' that Galileo's and Newton's true theory might provide as a means to determine and measure t r u e , i.e. *absolute* motions. The case is evident e.g. in the shift of formalism from constant Δt (Newton) to variable dt (Leibniz). As I see things, Cohen was only led into error when he abandoned his *first insight* into Newton's true concept of time in favor of Whiteside, and consequently in favor of the philosophy of Leibniz, Descartes, and all other antipodes of Newton, which philosophy is inextricably intertwined with the Leibnizian calculus, and thus means the philosophy behind the formalism of classical mechanics.

47. On p. 117-118, Cohen deals with Newton's third law, stating that this law „has often been a source of confusion.“ Unfortunately, Cohen adds to this confusion by alleging that two forces, equal in magnitude and opposite in direction, *could not produce* a condition of equilibrium if acting on different bodies (p. 117 below, 118 line 9). This allegation evidently contradicts Newton's explanations of the validity of the third law as given in the *Scholium* to follow Corollary 6. Cohen remarkably does not make use of this explanation, rather he tells the reader an insignificant anecdote about John Adams and Benjamin Franklin.

Newton, for instance, explains an example of the earth, „cut by any plane EG into two parts EGF and EGI [of different size]; then their weights toward each other will be equal [and consequently the unequal parts will *of course* stay in equilibrium] ... And if these weights were not equal, the whole earth ... would yield to the greater weight and in receding from it would go off indefinitely“ [„abiret in infinitum“] (Cohen-Whitman p. 428). The instance can be understood if one sees that the part EGF attracts the part EGI with a force which, according to Newton's def. 8, is given through the product of the relatively great quantity of matter in EGI with the relatively little accelerative force exerted by the smaller body EGF, whilst the part EGI attracts the part EGF with a force given through the product of the relatively little quantity of matter in EGF with the relatively great accelerative force exerted by the greater body EGI. As a consequence, the products 'small body EGF times great accelerative force exerted by EGI', and 'great body EGI times little accelerative force exerted by EGF', i.e. the weights of the unequal bodies relative to each other, are equal, and stand *of course* in equilibrium, to spite Cohen once more.

48. On p. 119-122, Cohen presents at length a consideration if „the concept of energy and its conservation appear in the *Principia*“ (p. 119). Correctly he refers to Peter Guthrie Tait who, in the middle of the 19th century, expressed the view that Newton's term „actio“ as it is used in the *Scholium* to follow the laws of motion (after Corollary 6, Cohen-Whitman p. 428-430) corresponds to the later on developed concepts of „work“ and „energy“ of classical mechanics. However, Cohen alleges that „the reckoning of 'action' by 'force and velocity

jointly' is not a feature of the *Principia*" (p. 119), and this happens on false grounds. It happens because Cohen, using a mistaken concept of 'force', falsely translates Newton's definition of 'actio' (given through „force and velocity jointly", i.e. [impulsive] *force times velocity*), and consequently cannot see that this product exactly yields a magnitude 'mass times velocity squared' as an equivalent to the (kinetic) 'energy' of classical mechanics $E = mv^2/2$ (Max Jammer has shown that the '1/2' was only added at will in 1829 by Coriolis, for the sake of better integrating facilities). The line of reasoning of Tait shows clearly that he *refers to applied mechanics*, that is to motion against any sort of resistance, as it is the general concern of Leibnizian 'dynamics'. This concern, as an exception of his general issue 'unresisted motion' in book 1, is Newton's too, in his talk on „the effectiveness and usefulness of all machines or devices" at the end of the said *Scholium.* Says Newton: „But my purpose here is not to write a *treatise* on [applied] mechanics" (Cohen-Whitman p. 430 paragraph two), to settle that this applied mechanics which later on adopted together with the Leibnizian principles of 'energy' and 'work' the name 'dynamics' is *not* a general subject of his *Principia*, book 1.

The whole matter, if one considers correctly what Newton meant by „force and velocity jointly", shows clearly that Peter Guthrie Tait was right, and that Newton indeed knew under the name „actio" that magnitude which we today call 'kinetic energy', even though the concepts, say *the terms* 'work' and 'energy' were „developed only long after the *Principia* was published" (Cohen p. 119, line 8 from below). By the way, my analysis of Cohen's criticism of Tait helps to *really understand* why Newton, in the *Principia*, book 1 concerned with *unresisted* motion, did not develop here concepts of 'work' and 'energy' even though he knew these physical magnitudes very well.

49. On p. 129, Cohen draws extensively on D.T. Whiteside's „Mathematical Papers", quoting a phrase on „the logically tight exposition of the principles of motion under accelerative forces", which shows the wrong idea of both Cohen and Whiteside as if the *Principia* should deal with „accelerative forces", and with *only* such forces to generate the motions of bodies. Evidently, Whiteside as well as Cohen tacitly assumes that Newton's theory of force and motion deals with the very same subject as classical mechanics, ignoring Newton's repeated statement (as quoted above from def. 4, *Scholium* after def. 8, law 1 and law 2, and *Scholium* after Corollary 6) that „accelerative forces" such as the centripetal force, in the *Principia* are *only mathematically treated 'a s i f'* they could 'attract' and thus 'move' bodies, whereas t r u e motion is generated and changed o n l y by „impressed forces" that *spring off from* (not only) centripetal forces (def. 4). In this context, Newton's *Lemma X* should be quoted, where, in the Section 1 of „First and Ultimate Ratios", Newton for the first time makes use of the term „force", explicitly speaking of „any *finite* force" (Cohen-Whitman, p. 437, line 2 from below, my italics), which „force" being „finite" means certainly not an (infinitely!) accelerating, rather an „impressed force" as the only natural agent that is really able to „urge" bodies, as Newton here puts it.

50. On p. 130, at the end of paragraph three, Cohen refers to the just quoted *Lemma X* („lem. 10") in a misleading manner. In vain he tries there to understand Newton's teaching, because Cohen does not carefully consider Newton's words „vis finita", „vis determinata et immutabilis", „sive eadem continuo augeatur vel continuo diminuatur". If Newton had meant an *accelerative* force, why then should that force only „*ipso motus initio*" (as Newton puts it) stand „in duplicata ratione temporum"? Is not the „duplicata ratione temporum" the *general* measure of an accelerating force, no matter if „ipso motus initio" (i.e. „at the very beginning of the motion") or not? In fact, with respect to this „ipso motus initio" the *Lemma* can only be

understood if one adopts the view that the „vis finita“ which Newton explicitly speaks of, i.e. the *finite force impressed on a body, at the very beginning of motion only* can be calculated according to the known mathematical measure „duplicata ratione temporum“ of uniformly accelerated motion, and *of a supposed centripetal, continuously accelerating force*.

51. On P. 130, after paragraph three, Cohen should have introduced a hint to the most important *Scholium* which Newton presents after lem. 10. The *Scholium* shows how Newton understands the theory of proportions, and explicitly applies it to „quantitates indeterminatae diversorum generum“, that is *to variable quantities of a different kind.* Thus he makes use of a decisive improvement of that theory, extending it (following John Wallis's „Mechanica“ of 1670) to heterogeneous quantities or *incommensurables*, contrary to some limitations which during the Middle Ages had falsely been introduced into Euclid's „Elements“. Cohen refers to that subject matter only on p. 312.

52. On p. 146, paragraph two, Cohen repeats his idea „that Newton had never read Galileo's *Two New Sciences* [published at Leyden as early as 1638] and knew only some selected portions of Thomas Salesbury's English translation of the *Dialogue* on the two systems of the world“ [published at Florence as early as 1632]. Obviously, to put this somewhat extravagant idea against Newton's explicit reference to Galileo means a hobby-horse for Cohen. In my view, it should, in favor of Newton's clear words, be kept back, so long as there is only that „considerable evidence“ to which Cohen cautiously resorts, but no valid proof against Newton (I cannot, by the way, imagine how such a proof could ever be supplied).

53. On p. 149, paragraph two, Cohen deals with Newton's strictly mathematical, and explicitly non-physical treatment of centripetal forces, or forces of attraction, alleging that Newton nevertheless „believed in the existence of such forces.“ However, in the next phrase he refers to Newton's expressed rejection of „the idea that forces could act over enormous distances without 'the mediation of something else'.“ These inconsistent statements again result only from Cohen's superficial reading of Newton's text, and from the un-Newtonian view on „forces“ of classical mechanics. *Nowhere* in his writings does Newton express a belief in the reality of attracting forces. Quite the contrary, e.g. in def. 8 he *explicitly warns the reader* to think that he, when speaking of attraction toward a centre, would anywhere define „a species or mode of action or a physical cause or reason“, or that he were „attributing forces in a true and physical sense to centers (which are mathematical points)“ if he happens „to say that centers attract or that centers have forces“ (Cohen-Whitman, p. 408). Once more must be quoted against Cohen what Newton writes on *not centripetal and attracting*, but impulsive „impressed forces“ in def. 4, in the *Scholium* following def. 8, in the first and second law, stating that *these „impressed forces“ only* (which in the case of free fall successively spring off from centripetal force as but their source, see def. 4 and *Scholium* after Corollary 6) are able to generate motion respectively changes of motion in a body in a true and physical sense. As a consequence, it is very clear that Newton also rejected the idea of centripetal forces *acting at a distance,* as he already has said in def. 8 that the accelerative quantity of centripetal force is an „efficacy diffused from the center through each of the surrounding places in order to move *the bodies that are in those places“* (Cohen-Whitman, p. 407, my italics). The words put in italics indicate the picture of a 'field of force' as a local source of forces that are *locally* „impressed“ (!) on a body to make it move. Thus they cross the general false belief as if Newton, contrary to his explicit protest against such a concept, had ever believed in the reality of attracting *forces acting at a distance*.

One should note here that contemporary reviews of Newton's philosophy were well aware of these (immaterial) locally acting impressed forces, e.g. in the Acta Eruditorum of Oct. 1713, p. 444-448, where we read: „Admittit [Newton] igitur attractionem, *quae non sit actio materiae in distans, sed actio causae cujusdam immaterialis*, materiam perpetuo certis legibus moventis et regentis" (my italics). Samuel Clarke, taking Newton's part in his correspondence with Leibniz of 1715-1716, stressed the reality of immaterial entities (such as time, space, and forces) against Leibniz. Somewhat later it was James Clerk Maxwell who, about 1840, discarded the 'classical' idea of forces 'acting at a distance' in favor of locally acting 'fields of force'. Maxwell expressed the view that the false action-at-a-distance-concept could *not* rightly be attributed to Newton („On actio in distans", Proceedings of the Royal Inst., Feb. 1873).

54. On p. 149-153, Cohen once more explains his idea of a special „Newtonian style" which Newton should have used, advancing „from the mathematical construct of a single mass point moving about a mathematical center of force to a system of two interacting such masses" (p. 150, line 9 from below), that is to a stage „in which the center of attraction will no longer be a mathematical point, but rather a second body" (p. 151 paragraph two). However, in contradiction to Cohen's statement one must see that *nowhere* in the *Principia* appears something like a „mass point" which, with respect to Newton's concept of mass as 'quantity of elementary material particles in a body', would mean a contradiction in terms. The concept of a 'point mechanics' belongs to classical mechanics only (i.e. to the mechanics developed on Cartesian-Leibnizian foundations by d'Alembert, Euler, and Lagrange in the 18th century). Note that even the „mathematical center of the force" (Cohen) according to Newton is not a „mass point", but rather a *mathematical* point (cf. Cohen-Whitman p. 408, line 16). Moreover, contrary to Cohen, it is not true that Newton, in sec. 11, should advance to a stage „in which the center of attraction will .. be [no longer a mathematical point, but] .. a second body", rather Newton shows here that attraction between two bodies must be understood as *mutual* („attractio mutua"), and consequently it is not directed to (the centre of) a body, but rather to a *common centre of gravity* („gravitatis centrum commune") *apart from the centres of the bodies*, which common centre of centripetal force is exposed in sec. 11 *as a mathematical point in free space again*.

55. On p. 151, paragraph three, Cohen accuses anonymous „scientists and philosophers [to] have not been willing to accept Newton's statements about the mathematical level of discourse at their face value." This accusation reflects on Cohen himself, who, even calling Newton's philosophy by the Leibnizian term 'dynamics', in numerous places ignores Newton's words in favor of interpretations which reduce it to a materialist science based on Cartesian-Leibnizian foundations. As to „the mathematical level of discourse", Cohen again and again ignores that Newton is very explicit in stressing *his view of 'centripetal forces' only* as a purely mathematical concept, and so does Newton once more in the introduction to sec. 11 to which Cohen refers here. Nowhere, for instance, did Newton ever call into question the real physical state of „impressed forces" as defined in def. 4, and cited in the first and second law. Quite the contrary, Newton explicitly states *these „impressed forces" alone* are able to generate or change true motion (see Cohen-Whitman p. 412, line 11), and *that „centripetal force" does not act by itself*, rather as a sort of 'source' where „impressed forces" generate from, which finite impressed forces, then, as really acting causes of motion generate step by step finite impulses to form in the limit a continual accelerated motion, as it is described with reference to the accelerated motion of free fall in the *Scholium* to follow Corollary 6: „When a body falls, uniform gravity... *impresses equal* [finite] *forces* upon that body and generates [not uniform acceleration, but] *equal* [finite] *velocities* [!]..." (Cohen-Whitman p. 424, line 6-8, my

italics). The same step-by-step-generation of change of (direction of) motion can be seen in the figure to illustrate Newton's *proposition I* referring to circular motion according to a centripetal force (Cohen-Whitman p. 444). Thus Newton makes it absolutely clear that his discourse only is restricted to a mathematical level in so far as „centripetal forces" are treated in the *Principia, treated* a s i f they, without mediating „impressed forces", could by themselves immediately produce uniformly or continually accelerated motions.

The known objections of Newton's materialist contemporaries against universal attraction, to which Cohen refers on p. 153-154, then, did not meet especially the *accelerated* mode of motion, but rather the evident *immateriality* of Newton's acting forces which, however, were often (and by Cohen too) mixed up with centripetal forces, against Newton's repeated warnings. In the course of the 18th century the case was only settled after scientists accepted the materialist idea of centripetal force not only as a mathematical model, but, as *a real faculty of the mass* of central bodies to „stretch out through enormous reaches of space" (Cohen p. 154 paragraph three) in order to instantaneously grasp and 'attract' distant bodies. And this un-Newtonian materialist concept (which, as 'mass attraction', means a basic concept of classical mechanics and modern physics) is constantly present behind Cohen's considerations, and determines and misguides his interpretation not only of Newton's restriction to mathematics in the case of „centripetal force", but also of the *Principia* in general .

56. On p. 153, paragraph three, Cohen interprets the beginning of book 3, making Newton say that he, in the preceding two books, had been [generally] setting forth principles which „are not, however, philosophical but strictly mathematical." As a consequence of Cohen's abstinence from philosophy, his aim is to separate Newton's „Philosophia naturalis" from philosophy in general, by interpreting its „principles" as if these were purely *mathematical* principles. Newton, however, says: „In Libris praecedentibus Principia Philosophiae tradidi, non tamen Philosophica sed Mathematica tantum, ex quibus videlicet in rebus Philosophicis disputari possit. Haec sunt motuum et virium leges et conditiones, quae ad Philosophiam maxime spectant." The first phrase shows that Newton speaks of „Principia *Philosophiae* Mathematica", i.e. of mathematical principles *of Philosophy*, and not of *purely mathematical* principles. In the second sentence Newton states that he, in book 1 and 2, had expanded the (mathematical) laws and conditions of motions and forces *which have a special meaning for philosophy.* One sees clearly, then, that Newton's aim is always at philosophy, and that he uses mathematics as *only a tool* for his philosophical investigation of nature. Contrary to Cohen's allegation, nowhere does Newton insist in books 1 and 2 that „he had been [generally] exploring 'principles' only mathematically and not physically", because, whenever Newton expresses something in that direction, he clearly refers *to his restricted mathematical treatment of 'centripetal' forces only.*

57. On p. 154, paragraph two, Cohen quotes from Voltaire's „Lettres philosophiques" a phrase which would have Newton say that he used „the word attraction only to express ... *a quality inherent in matter*..." (my italics), and Cohen assumes that Newton „would have agreed."

In contradiction to that statement, Newton tells us in the *Principia*, book 3, Rules for the Study of Natural Philosophy, rule 3, that he is „by no means affirming that gravity is essential to bodies. By inherent force I mean only the force of inertia" (Cohen-Whitman, p. 796 line 9-8 from below). Thus the reader can see that Newton, against Cohen's assertion, would never have agreed with Voltaire's statement on attraction to be „a quality inherent in matter".

58. On p. 158, paragraph 4, Cohen refers to book 1, pro. 66, corol. 14 where Newton makes use of the term „absolute force“ (of an attracting body), „thus bringing in the mass of bodies.“ Again Cohen erroneously alleges here that „Up until sec. 11, Newton has been exclusively considering unit masses or mass points and so has used only ‘accelerative’ forces (or accelerative measures of forces).“ Cohen wants the reader to believe that „absolute force“ should mean that measure of force which differs from „only accelerative“ measures by the bringing-in of „mass“. Now, since these „accelerative“ or „motive“ quantities of centripetal force are defined by Newton in def. 7 and 8, whilst the „absolute quantity of a centripetal force“ is defined in def. 6, Cohen misguidingly insinuates a relation between this „absolute force“ and the „accelerative force“ with reference to the role of mass. Actually, the mass that determines the „absolute quantity of centripetal force“ means the quantity of matter *of the central attracting body* (see def. 6 referring to the instance of the „moles“ of a loadstone), whereas the mass that distinguishes the „attractive“ quantity of a centripetal force (def. 7) from its „motive“ quantity (def. 8) means the quantity of matter *not of the attracting*, but rather *of the attracted body.*

59. On p. 159, line 9, Cohen gives another nice example of his somewhat loose approach to Newton’s teaching, as he once more repeats his mistaken view as if Newton, in secs. 1-10, had „considered bodies to be *more or less* [sic] unit masses or mass points, *without ..mass...“* (my italics). The idea to speak of „unit masses or mass points without mass“, certainly a contradiction in terms, and of course also a contradiction to Newton’s definition of mass (def. 1), needs no further comment.

60. On p. 159, line 9-8 from below, Cohen, referring to Newton’s phrase that some proposition will be „of some use in philosophical questions“, continues, commenting “that is, in questions of natural philosophy, or physical science.“ - Clearly one can see Cohen’s aim here to shift the meaning of Newton’s *magnum opus* from essentially *philosophy* to essentially *physical science* in the modern sense. As the *Principia* meant a foundation of a *true philosophy* that stands „as a mighty fortress against the attacks of the atheists“ (Cohen-Whitman p. 398; Roger Cotes’s preface to the second edition), one understands on what grounds people from Newton’s days to those of I. B. Cohen again and again have tried to distil from the *Principia* any purely scientific contents apart from its philosophical and theological meaning and implications. Cohen’s „Guide“, as criticized above and in my letter of Aug. 29, in numerous points gives a warning example to show that he who wants to separate the scientist and mathematician Newton from the philosopher, must fail to understand the *meaning* of Newton’s words, first of all the meaning of the central word „force“, and consequently must hit upon un-Newtonian ideas and concepts, mostly, as we have seen above, on ideas and concepts which, even if expressed in a purely mathematical manner, inevitably carry the teachings of Newton’s philosophical antipodes Descartes and Leibniz.

.....Somewhat to my surprise, I found in Cohen’s essay on „Newton’s Second Law and the Concept of Force“ (in „The Annus Mirabilis of Sir Isaac Newton 1666-1966“, R. Palter ed., Cambridge/Mass. 1970, p. 149) the following phrase:

„We must be careful lest we bind Newton’s thinking in an intellectual strait-jacket that satisfies our own requirements at the expense of understanding his.“

One could not have said it better. Sincerely yours …..

Ed Dellian D-14169 Berlin, Oct. 16, 2000

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ISAAC NEWTON, THE PRINCIPIA
A New Translation by I. Bernard Cohen and Anne Whitman
preceded by a Guide to Newton's *Principia* by I. Bernard Cohen

Dear Sir/Madam,

with this letter I want to complete the criticism of Prof. Cohen's „Guide", opened with my letters of Aug., 8 and 29, and Sept., 7, 2000. I refer now to Chapter ten of Cohen's „Guide" (p. 293-368).

1. On p. 293-297, Cohen recommends to the reader „some useful commentaries and reference works as first guides to anyone who wishes to study *the mathematical and technical structure* of the *Principia*" (my italics). Obviously, Cohen has no idea that anyone might wish to study *the philosophy of nature* to which Newton explicitly has dedicated his *magnum opus.* Consequently, one cannot find any reference to works on this subject matter in Cohen's list, as for instance the correspondence with Leibniz edited by Samuel Clarke in 1717, followed by many commenting editions up to today (cf. e.g. H. G. Alexander, „The Leibniz-Clarke Correspondence", Manchester 1956); not to mention Maclaurin's „An Account of Sir Isaac Newton's Philosophical Discoveries" of 1748, Rosenberger's „Isaac Newton und seine physikalischen Principien" of 1895, Betty J. T. Dobbs's and Margaret C. Jacob's „Newton and the Culture of Newtonianism", New York 1998, etc. etc. Cohen's confinement to mathematics and technics, however, is not only a confinement, *but rather a corruption* of Newton's teaching, since the mathematical and technical structure of the *Principia* cannot be understood correctly without taking into consideration the philosophical principles lying behind, especially Newton's view on space and time, and of force and motion (of „cause" and „effect") and their measurement in units of space and time.

1) The mathematical structure of the *Principia* should clearly be called a *geometrical* structure, opposed to the algebraic-arithmetic character of Cartesian-Leibnizian mathematics that governed the development of classical mechanics after Newton, in the 18th and 19th century. Geometry, however, has been a *philosophical* matter not only since the times of Plato. For Newton, geometry means „that part of universal mechanics which reduces the art of measuring to exact propositions and demonstrations" (Cohen-Whitman p. 382), or, more correctly rendered, geometry, according to Newton, is that part of mechanics which *states and demonstrates the art of exact measurement itself, as the only means by the application of which philosophy comes up to a strict science based on demonstrations of truth.* Now, as measurement first and foremost means to measure distances (or lengths, or spaces) and times, the geometric art of measurement will certainly be determined by the geometer's philosophical view on space and time. Newton unveils this view in the *Scholium* on time, space, place and motion to follow def. 8 (Cohen-Whitman p. 408-415). Here we find Newton measuring variable spaces and variable times *relative to scales of absolute space and absolute time*, which method Galileo was the first to invent and publish, in his „Discorsi" of 1638.

Obviously, Newton measures spaces and times, velocities and motions like Galileo in relation to the structure of a space-time frame of reference, which structure is determined through the specific graduation of the said scales of absolute space and absolute time.

2) At a closer investigation one will see that the structure of this frame of reference of course is one of Euclidean shape. For Galileo and Newton as philosophers of nature in a Neoplatonic tradition, absolute space and absolute time mean absolute realities, i.e. real emanations of God. Variable spaces and times, velocities and motions of bodies then can be determined and measured definitely in relation to this realist Euclidean frame of reference. This frame of reference, say this arena of events, itself is metrically determined through a lattice constant given as a quotient of the elementary particles of space and time which particles, lined up in a row, form the said metric scales of absolute space and absolute time. This way of measurement, deeply rooted in a realist philosophical tradition, as it derives the measure of any quantity from an absolute scale, guarantees the reality of the measured quantity - even though its measure is a „relative“ one insofar as it is obtained *in relation to* (i.e. relative to) the absolute scale.

3) Most importantly, Newton, since he has an absolute space-time frame of reference at his disposal, is able to distinguish *absolute* or *true* motions of bodies from *only relative* motions, relative *with respect to other bodies (whose state of rest or motion is not known) serving as arbitrary systems of reference.* And this issue marks again one of the main differences between Newton’s theory of absolute motion and classical mechanics, which, as it is rooted in the relativist philosophy and theory of motion of Aristotle, Descartes and Leibniz, knows only relative motions to be measured in relation to other bodies (i.e. systems of reference), and consequently is based on the principle of equality of all optional systems of reference, i.e. of the arbitrary interchangeability of bodies of reference, as for instance the earth (with respect to the motion of the sun) or the sun (with respect to the motion of the earth). Accordingly, the theory of motion of classical mechanics contrary to Newton’s is not able to distinguish *true* motion from only *apparent* motion, as for instance in the case of whether the sun or the earth should move respectively. Newton, who aimed at and developed a theory of true motion, in some way demonstrates the eternal truth of the Copernican system of the world, a demonstration which is not within the scope of classical mechanics.

One should well notice that nothing of these considerations and explanations, not even the slightest hint at the meaning and the philosophical, cultural and scientific importance of Newton’s theory of *absolut*e motion as a proof for the truth of the Copernican system, can be found in Cohen’s „Guide“ (cf. Cohen’s uninformative way to mention Newton’s main concern „absolute motion“ only once and indirectly in passing a paper of R. Rynasiewicz, p. 108).

2. On p. 295-296, Cohen, accusing the late S. Chandrasekhar that he, in his „Newton’s *Principia* for the Common Reader“, *„disdainfully and cavalierly* [!] dismisses the whole corpus of historical Newtonian scholarship“, gives a striking example of how Cohen’s own ignorance as to the whole corpus of *philosophical* Newtonian scholarship might be estimated. Cohen, though he is right in criticizing Chandrasekhar’s presentation of Newton’s second law, here again is not able to show the reader why and in which respect this presentation misses the true contents of Newton’s law, since he himself doesn’t know better than to make definitely and uncritically use of Chandrasekhar’s (and classical mechanic’s) un-Newtonian concept „force equals mass times acceleration“ *himself,* e.g. on p. 104, 110, 117 fn. 16, 119.

3. On p. 309, paragraph 2, Cohen, referring to some „Lexicon Technicum“ (John Harris, London, 1704), alleges that Newton had „tended to use ‘ratio’ and ‘proportio’ rather interchangeably.“ As the cited Lexicon according to Cohen holds that *any ratio* between any two quantities „in respect of their Greatness or Smallness“ signifies a proportion, one can well estimate the limited reliability of that source. In fact, it would be hard work for Cohen to show where Newton in the concrete uses „ratio“ and „proportio“ *interchangeably*. Quite the contrary, at closer investigation one will mostly find him carefully distinguishing „ratios“ between *homogeneous* magnitudes from „proportions“ between *heterogeneous* ones. It is quite a mercy that Cohen *as a translator* has „followed Newton’s usage, rendering his *proportio* as ‘proportion’ and his *ratio* as ‘ratio’“ (as he tells the reader on p. 309). On p. 311, however, one can see Cohen to ignore Newton’s deliberate usage of the terms „ratio“ and „proportio“, as Cohen indiscriminately treats Newton’s way of expressing ratios as an „older form of stating proportions“ (line 10 from below).

4. On p. 317, paragraph 2, Cohen praises highly the work of D.T. Whiteside on Newton’s „Mathematical Papers“. However, much more important than that praise seems to me Cohen’s warning which the reader of Whiteside should always be aware of: „In D.T. Whiteside’s commentaries many of Newton’s arguments and results are translated directly into the [Leibnizian] algorithm of the [Leibnizian] differential calculus.“ Actually, this is the case to such an extent that even N. Guicciardini, in his „Reading the *Principia*“, on p. 2 confesses that „Paradoxically, the effect of Whiteside’s penetrating mathematical insight is that of suppressing diversity and plurality. From this point of view, he did not help me in finding an answer to my search for the mathematical methods of Newtonian natural philosophy.“ In my view, to render Newton’s geometry into the language of the Leibnizian calculus by necessity produces a corruption of Newton’s teaching in so far as Newtonian *constant* quantities (such as e.g. the velocity of generation of just nascent quantities, and the line-elements of spaces and times) are treated as *variables* in that language, which usage consequently destroys Newtonian proportions by reducing them to equations in canceling the constants of proportionality. The most prominent example for this corruption is given, when the second law of motion is altered and reduced from $F \propto \Delta(mv)$ to $F = ma$.

5. On p. 317-323, Cohen, according to his headline 10.8 on p. 317, refers to Newton’s Book I, prop. 6 as to „Newton’s Dynamical Measure of Force“.- Two things must be objected to this wording: First, Newton does not present a „*Dynamical* measure of force“ here, since he does not deal with „Dynamics“ at all in Book I, as I have already stated in my letter of Aug. 28, item nr. 1. Second: Newton does not present a „measure of force“ pure and simple, rather he explicitly develops a measure of a „centripetal force“ in the said proposition. Cohen’s somewhat loose terminology in the criticized headline results again from his ignorance as to Newton’s distinction between *different kinds of forces* to be differently defined in def. 3 (vis insita vel inertiae), def. 4 (vis impressa), and def. 5-8 (vis centripeta). I have already pointed to this main defect of Cohen’s „Guide“ in my letters of Aug., 28, and Sept., 7.

On p. 321, line 13, Cohen, in summing up his preceding considerations, states that „This is Newton’s dynamical measure of a force...It is a *dynamical* measure because it measures the force by its *dynamical* effect, the rate at which the action of the force causes the moving object to deviate from a linear inertial path...As has been indicated earlier, this measure is not strictly dynamical, because it does not involve the factor of mass“ (my italics). - To this the following must be commented:

1) Cohen's usage of the term „dynamical" is very confused and confusing. Not only that this Leibnizian term does not fit to Newton's theory of motion in free spaces, but also Cohen presents an empty, circular definition of a „*dynamical* measure" [of force] by its „*dynamical* effect". Actually, Newton measures the centripetal force *by an effect on the motion of a body*, as he generally does it in the *Principia*, and there is nothing specific in prop. 6 that would require the adjective „dynamical" here.

2) Cohen's last phrase referring to „the factor of mass" might easily raise the reader's confusion to a higher level, since he feels called upon to consider the meaning of a „dynamical measure not strictly dynamical." It suffices, however, to say that the whole phrase is useless, because the „the factor of mass" *of course is involved here*, since Newton refers to the *motion* of a *body*. Remember that „motion" is always „mass times velocity" according to Newton's def. 2, and „body" is always synonymous to „mass" according to Newton's def. 1. In the following paragraph, Cohen mistakenly alleges prop. 1 (like in prop. 6) to assume „that all bodies in motion have the same mass - or - which is equivalent - that they are of unit mass". In fact, prop. 1 as well as prop. 6 refers explicitly to „any bodies" (prop. 1 corol. 4; Cohen-Whitman p. 446, line 4), respectively to „any body P" (prop. 6 corol. 5; Cohen-Whitman p. 455, line 14) which „any" clearly means *different* bodies, i.e. *different* masses. The sense then is that the described law holds for *any* masses whatsoever, and not only (as Cohen's words suggest it to the reader) for equal or *unit* masses.

6. On p. 324 paragraph two, Cohen speaks of „the *instantaneous measure* of centripetal force taken from corols. 1 and 5 of prop. 6" (my italics). However, Newton nowhere makes use of the term „instantaneous" or of the concept of instantaneousness. Since the term and the concept in physics denote changes of state that should occur without any elapse of time, which unrealistic idea is thoroughly un-Newtonian, one should never use it as an adjective to explain Newtonian concepts such as that of „centripetal force", the more as the term truly belongs to Leibnizian conceptions. In the cited instance this adjective is both superfluous and misleading.

7. On p. 333, Cohen, with respect to Newton's prop. 32, once again sees „evidence that the *Principia* is not written in the style or manner of Greek geometry." The statement to occur repeatedly in Cohen's „Guide" once again gives an instance of Cohen's loose choice of words. Any careful analysis of the *Principia* shows that it is *of course* „written in the style or manner of Greek geometry", even if supplied by Newton's method of first and ultimate ratios of nascent quantities, „that is, on the limits of such sums and ratios" (Cohen-Whitman p. 441 line 6 from below). Newton, (not only) in the Scholium to follow Lemma XI, is explicit in emphasizing his reliance on (Greek) geometry, and in the *geometric* meaning of his supplied method (cf. Cohen-Whitman p. 440-443).

8. On p. 335, line 3, Cohen, speaking of „Newton's *seemingly* geometric language" (my italics), continues to insinuate that we could translate Newton's presentation „rather directly into the more familiar [arithmetic] algorithm of the Leibnizian calculus." Cohen's aim is to justify especially the manner of especially D.T. Whiteside to take Newton's geometric expressions as synonymous with differentials and integrals. Contrary to this intentions it has been shown for instance by N. Gucciardini that there lies a deep gap between the methods of Newton and Leibniz.

9. On p. 362, paragraph three, Cohen, under the headline „Newton's Measures" tells us that Newton in the *Principia* „uses both English and French units of measure." Is it the same author who, on p. 92, line 7 of the „Guide", alleges that „in the *Principia*, Newton is generally

not concerned with units or with dimensionality“ ? Is it the same author who, again on p. 92, line 22, in an authoritarian manner states that „the *Principia* sets forth a dimensionless physics“ ? The answer is that Newton of course, as he teaches a theory of measurement of forces and motions throughout the *Principia*, inevitably makes use of units and dimensions. However, the most abstract part of his work, that is book 1 on the theory of unresisted motion, is based on the most abstract possibility of measurement - that is, to measure variable spaces and times in units of absolute space and absolute time, or relative to absolute graduated „scales“ of space and time, as numerical multiples of the elements of „space“ and of „time“, and to measure bodies in units of mass, that is as numerical multiples of that element of matter, or elementary particle, which all material bodies are built of. Proceeding to more concrete tasks then, in book 2 and book 3, Newton passes over to making use of conventional systems of measure and units of his time, such as those „French and English units of measure“, to which Cohen on p. 362 refers. Unfortunately, Cohen gives not the slightest hint to this solution of the discrepancy between his statements on p. 92 and on p. 362 of the „Guide“. Cohen’s reader, then, gets no information on the conditions of the validity of conventional systems of measure such as „French and English units“, which validity rests *on their proportionality* to the most natural system of measurement in units of space, time and matter. And, even more regrettably, the reader is left without any knowledge of to which extent Newton’s philosophy of nature, contrary to the Leibnizian continuum theory of motion, implies a most up to date *quantum theory of space and time, of force and motion* which even allows for the solution of an extensively disputed issue of modern physics in a strictly Newtonian *quantum theory of gravitation*. Useless to say that Cohen’s defective treatment of this central question, instead of bringing the true Newton to light, continues to bury the truth on space and time, on force and motion under a heap of paper, to the sustained damage of true science and philosophy, and of truth itself.

Conclusion

10. On p. 369, in his „Conclusion“, Cohen calls the *Principia* „a book of mathematical principles applied to nature insofar as nature is revealed by experiment and observation. As such, it is a treatise based on evidence.“ - To this general judgement, one must object what follows.

1) The *Principia* is not one book, but a sum of three very different books which, however, are systematically connected with each other. Book 1 „On the motion of bodies“ is a book on the true or absolute motion of bodies in time and *in space free of any resistance*, and on the causes („forces“) of motion, based on principles that are derived from experience and written in the language of geometry. Book 2 „On the motion of bodies“ is a book on motion *in resisting media* (i.e. air or water), which shows how the very abstract principles of book 1 nevertheless apply to concrete physical problems. Book 3 „On the system of the world“ is a book which shows how the theory of force and motion of books 1 and 2 generally can be applied in order to explain several phenomena of nature such as the motions of the celestial bodies and of the sea.

2) Newton’s principles are derived *from experience, not „revealed by experiment“*. Of course Newton made skilful experiments, but not to yield „evidence“, as Cohen assumes, but to deduce (!) real *experience* from the great variety of phenomena. Besides providing a source of experience, experiments for Newton served not as a means of proof, but as actual *confirmations* of results first obtained theoretically and demonstrated by mathematical application of the geometric principles.

3) In book 1, Newton is deeply devoted to a Neoplatonic view of „nature“, i.e. the view of „nature“ *as something eternally real and true that lies behind the phenomena*: absolute time and absolute space as some origin (present emenations of God) from which measurable times and spaces descend; centripetal forces as some source from which impressed forces spring off in order to cause the motions of bodies. „Nature“ in this context means *natura naturans*, the sum of all generating principles, ultimately referring to the creator himself. In books 2 and 3, Newton refers to *natura naturata*, i.e the sum of all created things, or the creation itself („mundus“). Cohen's use of the term „nature“ shows that he does not know anything about Newton's Neoplatonic background, and about the philosophical difference between the said concepts of „nature“.

11. The rest of Cohen's „Conclusion“ from p. 369 to p. 370 (as many of his preceding considerations) is sketchy, unsuitable and unreasonable, not worthy of further critical investigation. Generally spoken, in my view Cohen's aim was to put definitively an end to all that newer research on Newton, mainly of scholars with philosophical interests, which rose in the last five decades, in consequence of what Sir John Maynard Keynes had published in 1946, as a sum of his investigation of some unknown papers left behind by Newton, called the „Portsmouth Collection“. Up to then, people had understood Newton as a harbinger of the age of reason, and had interpreted his work as a cornerstone of modern rationalism in the sense of classical mechanics. But Keynes published a very different view, calling Newton „not the first of the age of reason. He was the last of the magicians, the last of the Babylonians and Sumerians, the last great mind which looked out on the visible and intellectual world with the same eyes as those who began to build our intellectual inheritance rather than ten thousand years ago“. Maybe I. B. Cohen aimed to restore the traditional rationalist view on Newton's *magnum opus* with the best of intentions. I wanted to show, however, that this could not be done but at the expense of authenticity and truth.

Sincerely yours

Ed Dellian