# Newton on Mass and Force.

## A Comment not only on Max Jammer's "Concepts of Mass" (1961; 2000).

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## Abstract

The concepts of mass and force, as understood in contemporary physics, mean qualities of matter. The somewhat mysterious quality "mass" is said to appear in two or even three different ways: inertial mass, active gravitational mass, and passive gravitational mass. Generally Isaac Newton is taught to have first introduced the concept and its different appearing as inertial or gravitational mass into theoretical physics, while Albert Einstein is highly praised for having shown the indiscriminate equivalence of both concepts. A look into Newton's *Principia* of 1687, however, helps to see that Newton neither understood "mass" as a *quality*, but rather as only *another name* for a *quantity of matter* (which *quantity* he defined not in words, but mathematically), nor did he ever explicitly or implicitly teach any distinction between an "inertial" and a "gravitational" aspect of matter. So it seems that to rely on Newton's authentic teaching could perhaps not only relativize Einstein's merit as to the simplification of the concept of mass, but also make proof against Jammer's (2000) depressing conclusion "that the notion of mass, although fundamental to physics, is still shrouded in mystery." Accordingly, the paper aims at showing some quite surprising insights into the realm of modern physics - from the Newtonian view on the closely related, but different entities "mass" and "force".

Les concepts de la "masse" et de la "force" comme entendus dans la physique contemporaine, signifient qualités de la matière. La qualité "masse", mystérieuse et pas comprise, apparaît en deux ou jusqu'à dire trois formes: masse inerte, masse gravitante active, et masse gravitante passive. Selon la doctrine générale, Isaac Newton introduit le concept et ses différents inertes et gravitants aspects dans la physique théorique, contre quoi Albert Einstein est glorifié pour la démonstration de l'équivalence indiscernable des ces deux aspects. Il es vrai que jeter un coup d'œil dans la "Principia" du Newton (1687) informe de ce que Newton ni comprendrait la "masse" comme une *qualité*, mais seulement comme *une autre dénomination d'une quantité de la matière* (quelle *quantité* il définît pas du tout littéralement, mais mathématiquement), ni il jamais enseignait explicitement ou implicitement quelque distinction entre une qualité "inertielle" ou "gravitante" de la matiêre. En conséquence, il paraît que se rapporter à la doctrine authentique du Newton, peut rendre en état de non seulement relativiser le mérite de l'Einstein concernant la simplification du concept de la "masse", mais aussi immuniser contre Jammer's (2000) conclusion depressive "*that the notion of mass, although fundamental to physics, is still shrouded in mystery."* Dès lors le papier présente quelques découvertes assez surprenantes dans la règne de la physique moderne - avis de Newton sur les entités liées étroit, mais toutefois differents - la "masse" et la "force".

## Key Words

*Newton's "Principia" - the key to understand "mass", "force", and more.*

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### I Introduction

The concepts of "mass" and "force" as basic constituents (next to "space" and "time") of the *quantitative* and therefore *mathematical* (i.e. *geometrical)* new Natural Philosophy of the seventeenth century began their career with Sir Isaac Newton's "Principia mathematica philosophiae naturalis" of 1687. Among philosophers and scientists this fact is well known to the same extent to which the contents of Newton's book is unknown, or at least is not sufficiently understood. The difficulties to understand Newton's authentic theory result not only from the fact that the book was originally composed in Latin. Rather philosophers shy away from its mathematical appearance, seeing the book not as a work in philosophy but in science. To the even greater confusion of physicists, however, many concepts of classical mechanics, which are generally attributed to Newton, at closer inspection cannot be found in his geometrical theory of motion of bodies, which is the main subject matter of his *Principia*. For instance, the law of "force" and mass-acceleration ("force equals mass times acceleration", $\boldsymbol{F} = m\boldsymbol{a}$) which classical mechanics is based on, though generally known as "Newton's second law of motion", is not a true representation of Newton's law, and of his concept of "force". Historians of science do know that Newton's law differs substantially from $\boldsymbol{F} = m\boldsymbol{a}$, since E. J. Dijksterhuis in 1950 reported this finding[2]. Max Jammer, who in his "Concepts of Force" of 1957[3] did not yet mention Dijksterhuis, mean-while reluctantly pays some tribute to this new view when he, in his "Concepts of Mass" of 2000, repeatedly refers to $\boldsymbol{F} = m\boldsymbol{a}$ as to "Newton's second law *in Euler's formulation*" (pp. 5,12,17; my italics). As a matter of fact, it was Leonhard Euler who, in his "Mechanica" of 1736, developed this formula without making any reference of or giving any credit to Newton, and presented it to the Berlin Academy of Sciences uncontradictedly as *his own* "Découverte d'un nouveau principe de méchanique" in the year 1750[4].

What has been said above of Newton's concept of "force" in the second law is as well true with respect to some other basic concepts of classical mechanics, as for instance the concept of "kinetic energy" which stems from Newton's philosophical antipode G. W. Leibniz. After having concentrated on the study of the *Principia* for more than twenty years (I published a selected edition of it, translated from Latin to German, in 1988, as “Mathematische Grundlagen der Naturphilosophie” [5]), I dare to say that Newton’s authentic theory of motion has nearly nothing to do with that “classical mechanics” or so-called "Newtonian dynamics", which latter term unfortunately confuses Newton's name with the neo-Aristotelian term "dynamics" that G.W. Leibniz created in 1695 as a designation of his own anti-Newtonian theory, in his "Specimen dynamicum pro admirandis Naturae Legibus circa corporum vires et mutuas Actiones detegendis et ad suas causas revocandis".

I will prove my assertion concerning the difference between Newton's concepts of "force" and "mass", and those of classical mechanics, by concentrating on the origin of the concepts of “mass” known in classical mechanics: inertial mass, and gravitational mass, and I shall follow Max Jammer’s presentation of these concepts in his second book on mass (COM 2000).

## II Some remarks on Newton’s *Principia* and his definition of mass.

If one studies Newton’s writings, one is soon impressed by seeing how carefully the author chose his words. The *Principia*, certainly Newton’s masterpiece, presents itself as a most stringent composition of arguments, which cannot be read but in the way one is used to read codes of law (at least in Germany). Newton did not write for the common reader. Therefore those who try to read his book as if it were some general introduction to physics cannot understand him. Newton teaches not simply physics, but “Mathematical Principles of Natural Philosophy”. Moreover, he implicitly presupposes the teaching of Galileo Galilei[6] who, in his “Discorsi” of 1638, laid the ground for the exact geometrical measurement of motions in space and time. And Newton, as well as Galileo, composed his treatise on motion as a *strictly geometrical* art[7]. Consequently the usual attempts to understand Newton by rendering his geometrical arguments into the arithmetic language of Leibnizian analysis (as it was first done by Leonhard Euler in 1736) must necessarily miss the point[8].

What does Newton say about “mass”? Let us read the “Definitio 1” which opens the *Principia*:

> “Quantitas materiae est mensura eiusdem orta ex illius densitate et magnitudine conjunctim.” That is: *The quantity of matter is that measure of it which arises from its density and volume conjointly.*

The term “mass” does not appear in this definition until we read the explication, which follows immediately. Says Newton:

> “Aer densitate duplicata, in spatio etiam duplicato, fit quadruplus; in triplicato sextuplus. Idem intellige de nive & pulveribus per compressionem vel liquefactionem condensatis. Et par est ratio corporum omnium, quae per causas quascunque diversimode condensantur. Medii interea, si quod fuerit, interstitia partium libere pervadentis, hic nullam rationem habeo. Hanc autem quantitatem sub nomine Corporis, vel Massae, in sequentibus passim intelligo. Innotescit ea per corporis cuiusque pondus: nam ponderi proportionalem esse reperi per experimenta pendulorum accuratissime instituta, uti posthac docebitur.”
>
> That is: *If the density of air is doubled in a space that is also doubled, there is four times as much air, and there is six times as much if the space is tripled. The case is the same for snow and powders condensed by compression or liquefaction, and also for all bodies that are condensed in various ways by any causes whatsoever. For the present, I am not taking into account any medium, if there should be any, freely pervading the interstices between the parts of bodies. Furthermore, I mean this quantity whenever I use the term ‘body’ or ‘mass’ in the following pages. It can always be known from a body’s weight, for – by making very accurate experiments with pendulums – I have found it to be proportional to the weight, as will be shown below.*

Newton’s definition explicitly is one of “quantitas materiae”. This *quantity of matter* is the *subject of Newton's definition,* and the definition he gives for it is a *quantitative, not a semantic one*: “Quantity of matter” *is defined through a quantitative, mathematical term*: it is the quantity that is represented by *the product of* “density times volume”. And Newton makes this definition compulsory, saying *"I mean this quantity whenever I use the term 'body' or 'mass' in the following pages."*

Beyond this definition, however, Newton also describes a way how to determine this quantity *experimentally, namely by means of measuring a body’s weight* to which the quantity of matter is always proportional, according to Newton’s report on his pendulum experiments.

One should note again that Newton introduces the term “mass” only when he, in the quoted explication, explains that he, in the following, will make use of the terms “body” or “mass” in order to give “quantity of matter” *a (shorter) name*.

So, contrary to nearly everything that has ever been written on Newton’s *definitio 1*, this definition is n o t one of “mass”, nor is it circular, as some have opined; rather it is a formula for the quantitative determination of an even experimentally measurable quantity of a body’s material contents. The term “mass”, as it appears in Newton’s explanation of the definition, is clearly meant as only *another name* for this “quantity of matter”, as well as “body” (according to Newton) is such another name. *It is nothing but a different semantic expression for the subject of Newton's definition.*

Consequently, if we ask for the meaning of “mass” in Newton’s theory, we must infer that it precisely means “quantity of matter”, and nothing else. To be sure, this understanding refers us to the question what is “quantity of matter”. But, if scientists had focussed their interest on this subject, they would never have been led astray to investigate "mass" as some mysterious *quality* of bodies, as it happens in the ongoing hopeless endeavour to understand "the nature" of “mass”[9].

As a result we can see that it is not true what e. g. Max Jammer, in COM 2000, says at the very beginning of his book, when he asserts, “Isaac Newton began his *Principia* with a definition of mass” (p. 5).

In the same way, we may look at what Ernst Mach wrote in his book “Die Mechanik in ihrer Entwicklung” (Prag 1883)[10] in order to criticize Newton’s definition as circular. If we carefully distinguish Newton’s *subject of definition* “quantitas materiae” from its only semantic synonymous designation by the term “mass”, we understand that Mach, after incidentally having rejected this subject, like Max Jammer concentrates on the erroneous idea as if Newton had defined “mass”. Moreover, only by additionally imputing that Newton’s term “density” would *presuppose* a notion of “mass” which had to be defined first, Mach obtained the basis for his criticism**.** It is true, though**,** that the meaning of the term "density" is not self-evident; but in the context of Newton's definition of "quantitas materiae" it is quite clear that "density" means the number of material particles per a body's volume[10a].

As a matter of fact, at the background of Mach's attitude, as it comes to light in his unwarranted rejection of Newton’s subject of definition, there lay only his explicit anti-atomistic conviction[11]. Nothing forced him to reject or ignore Newton's (say the atomist’s) concept of ordinary matter to be a quantized quantity of elementary particles (i.e. to have a discrete structure), except his strong belief in the mechanics of his time, which (as we know better today) mistook matter for *a continuum* (the so-called *mechanics of the continuum).* Of course, this continuum mechanics was not established on Galileo’s and Newton’s atomistic philosophy of nature, but rather on Leibnizian and Kantian philosophical principles[12].

**III On quantities and qualities.**

Obviously the term “mass” *as such* does not convey any information about its meaning, in contrast to the term “quantity of matter”. In philosophy, *quantity* has been known ever since as a notion determined by number. A *quantity* of something then means a certain number or multitude of equal elements that belong to the same category. In contrast to this meaning of *quantity,* the term *quality* of something refers to some special characteristics of it that determine its individuality. Clearly Newton’s term “quantitas materiae” which is defined mathematically as the product of density and volume means a certain number of equal particles, or atoms (in the sense of the Ancients), or *quanta* of matter, and does not intend to characterize some special *quality of matter*.

On the other hand, when he decided to call this *quantity of matter* by the name of “mass” for the sake of brevity, Newton simultaneously introduced a *semantic definition* of the term “mass”: According to this authentic definition, mass, in the context of Newton’s true theory then should always exactly mean “quantity of matter”, thus unmistakably hinting at the quantization of all physical objects related to matter, and at his philosophical atomism. Had physicists kept to this subject instead of dealing with the philosophers’ *continuum*, physics would certainly have emerged as *quantum physics* from the beginning.

Today, in spite of Newton’s clear words, however, or should I better say*: in flagrant contradiction to Newton’s words*, physicists all over the world agree in accepting Newton’s term “mass” as an elementary *quality* of matter. Already in 1896, e. g. the French scientist and politician Charles de Freycinet, in his "Essais sur la philosophie des sciences", even used “mass” as that proper quality to define, i.e. to identify “matter”, saying *matter is all that which has* *mass* [13]. In a representative volume edited on behalf of the year 2000 as “Jahr der Physik” by the German Physical Society and the German Ministry of Education and Research, one article defines “the concept of mass” as describing *a quality of bodies which on earth is also called weight*. Dealing with the research work for this quality, the author very promising comes to this final conclusion:

*Several thousands of physicists and technicians from many countries of the world work together on accelerators and on their experiments. Their enthusiasm and very successful engagement make it appear realistic that we ‘in the near future’. i. e. within about 10 years – will understand the origin of mass*[14].

How could the obvious shift from quantity to quality happen? Did it happen in the course of time, as a result of the increase of knowledge? Did it happen as a result of scientific progress? Had Newton’s teaching on quantity of matter and on “mass” to be corrected according to new findings, or to experimental data? Were, perhaps, Newton’s pendulum experiments disproved?

The answer to all that is No. The shift from Newton’s notion of “mass” as merely another name for “quantity of matter” *to a quality of it* happened partly on grounds of insufficient study of Newton’s theory, partly due to philosophical prejudices concerning the "continuum", which prejudices supported the above analysed misinterpretation of Newton’s definition 1. This assertion can be proved quite easily through an investigation of the philosophy of Leibniz, and of the science of Leonhard Euler and the philosophy of Immanuel Kant who drew very much on Euler’s “Letters to a German Princess”[15] of 1769. As far as Kant’s philosophy of nature is concerned, I refer to Kant’s only poorly known „Metaphysische Anfangsgründe der Naturwissenschaft” of 1786[16] in order to show where e. g. Ernst Mach’s belief in the continuum theory of matter is based on. It is a matter of fact that the mathematician Leonhard Euler, and the German philosophers Leibniz and Kant had by far more influence on the foundation of classical mechanics than is commonly known. In general, they all maintained, and introduced into the theory of mechanics to the best of their abilities, a dogmatic, traditional, antiatomistic belief in the unstructured continuity of matter all over the universe, as it had been taught by Aristotle in older, and by René Descartes in newer times.

If considered from the Newtonian point of view, the said shift in the meaning of “mass”, as it was never caused through better knowledge based on experience, meant nothing but a *serious substantial corruption of Newton’s authentic teaching.* Actually it was and still is a corruption, which, together with others, called forth that immense mass of paper that buried the true theory of Newton. Had only Newton refrained from unnecessarily introducing in the *Principia* the word “mass” as an abbreviation of “quantity of matter” to his careless readers, his theory would have remained the very same, not affected, however, by all those sophisticated concepts of “mass”, and would not have been corrupted, and “shrouded in mystery” (Max Jammer, COM 2000 p. ix) as to this empty word’s possible meaning.

In the following, I will concentrate on showing how this corruption of Newton’s teaching was and still is responsible for that mysterious haze which still surrounds the concept of mass. For the moment, I want to sum up the result of my considerations concerning the different appearances of mass in contemporary physics – inertial mass, $m_i$, active gravitational mass, $m_a$, and passive gravitational mass, $m_p$ - by stating that all of this would never have appeared had only scientists and philosophers understood and respected the true authentic theory of Isaac Newton.

As to which extent even a man like Immanuel Kant thought to be able to authoritatively correct Newton at will (without any empirical foundation and evidence!), can be seen in the subsequent paragraph on inertia, that is on Newton’s “materiae vis insita”, as defined in the *Principia*, definitio 3: i.e. the *force innate in matter* - which also became corrupted, and converted into no longer a “force”, but into – well, *a quality of mass*, say *a quality of a quality.*

## IV On “inertial mass” $m_i$.

Newton, in the *Principia*, definitio 3, is absolutely explicit in attributing “inertia” to matter *not* as a quality, but as a *force implanted* (Latin “vis insita”) in matter, that is: as a physical entity in its own right which is *associated* with matter like some plant is associated with the soil wherein it is rooted, and this force is responsible for some special phenomena of material motion:

“Materiae vis insita est potentia resistendi, qua corpus unumquodque, quantum in se est, perseverat in statu suo vel quiescendi vel movendi uniformiter in directum.” Which means:

*There is a force implanted in matter having the power to resist, by which every body, so far as it is by itself, remains in its state of rest or of uniform straight lined motion.*

Newton's explanation to follow this definition further clears up the state of that "implanted force". It reads:

> "Haec semper proportionalis est suo corpori, neque differt quicquam ab inertia massae, nisi in modo concipiendi. Per inertiam materiae fit, ut corpus omne de statu suo, vel quiescendi, vel movendi, difficulter deturbetur. Unde etiam vis insita nomine significantissimo vis inertiae dici possit. Exercet vero corpus hanc vim solummodo in mutatione status sui per vim aliam, in se impressam, facta; estque exercitium illud sub diverso respectu & resistentia & impetus: resistentia, quatenus corpus ad conservandum statum suum reluctatur vi impressae; impetus, quatenus corpus idem, vi resistentis obstaculi difficulter cedendo, conatur statum obstaculi illius mutare. Vulgus resistentiam quiescentibus & impetum moventibus tribuit: sed motus & quies, uti vulgo concipiuntur, respectu solo distinguuntur ab invicem; neque semper vere quiescunt, quae vulgo tanquam quiescentia spectantur."

> That is: *The force implanted in matter is always proportionate to the respective body, and it does not differ at all from the inertia of the mass except in the manner it is conceived. Because of the inertia of matter every body is only with difficulty put out of its state either of resting or of moving. Consequently, the implanted force may also be called by the very significant name of force of inertia. Actually, however, the body exerts this force only when changing its state caused by another force that is impressed upon it, and this exertion is, depending on the viewpoint, resistance or impetus: resistance, insofar as the body, in order to maintain its state, strives against the impressed force; impetus, insofar as the same body, yielding only with difficulty to the force of a resisting obstacle, endeavours to change the state of that obstacle. Resistance is commonly attributed to resting bodies and impetus to moving bodies; but motion and rest, as commonly conceived, are distinguished from each other only by point of view, and bodies commonly regarded as being at rest are not always truly at rest.*

The Newtonian force implanted in matter, however, meant an offence to some enlightened adherents of the Cartesian philosophy, which did not allow for such things like "forces" as independent natural entities. It was e. g. G. W. Leibniz who, in his controversy with the Newtonian Samuel Clarke, fiercely rejected Newton's concepts of "forces of nature", accusing Newton of *having fallen back into the realm of darkness*[17]. Consequently Immanuel Kant, whose philosophy of nature is much more indebted to Leibniz than to Newton, made up his mind to clean mechanics from what he mistook for wrong, and claimed that, "in spite of its inventor's famous name", the concept of "vis inertiae" should be abandoned from science[18]. In general, scientists and philosophers of the 19th century followed the ideas of Kant, and reduced Newton's force of inertia *to a quality of matter* much in the way it was done to Newton's term "mass". It was again the Kantian Ernst Mach who, in his most influential book of 1883, insisted on attributing "forces" always strictly to matter as proper qualities thereof [19], in the same way as many others had already done it, and did (Mayer, Helmholtz, Kirchhoff, Hertz).

It is only a result of all these endeavours towards cleaning mechanics from odious Newtonian notions (odious if seen from the Leibnizian-Kantian philosophical viewpoint of e. g. Ernst Mach) that we are confronted today with the concept of "inertial mass" which is the subject matter of Max Jammer's COM 2000, chapter one. By relying on Newton's clear words we have understood now that Max Jammer is mistaken when he asserts that "it is Newton who has to be credited with having been the first to define the notion of inertial mass" [20]. Actually, no such definition as part of Newton's true theory of motion can be found in the *Principia* or elsewhere in Newton's writings. The understanding of inertia *as a quality* of matter, or mass, stems from the philosophy of Leibniz and Kant, and from their rejection of Newton's "force" of inertia on only philosophical grounds, as Jammer correctly has stated it in his COM 1961 (p. 88/89).

## V On "passive gravitational mass" $m_p$.

As it has already been noticed above, Newton took his pendulum experiments for the proof that "mass", i.e. the *quantity of matter* in a body, can be measured as it is proportionate to the body's weight. Sometimes scientists understand this proportionality of "quantity of matter" viz. mass, and of "weight" as an indication that already Newton implicitly had made use of two different concepts of mass, one "inertial mass", the other (passive) "gravitational mass" to appear as a body's weight [21].

Now, for Newton, "weight" clearly means a "force" again, as we can see it in his *Principia*, definitio 8: "Vis centripetae quantitas motrix est ipsius mensura proportionalis motui, quem dato tempore generat." That is*: The quantity of the centripetal force is the measure thereof, which is proportional to the motion generated in a given time.*

In order to understand what this has to do with weight, we must go into the first paragraph of Newton's subsequent explanation, which reads:

"Uti pondus majus in majore corpore, minus in minore; & in corpore eodem majus prope terram, minus in coelis. Haec quantitas est corporis totius Centripetentia, seu propensio in centrum, & (ut ita dicam) pondus; & innotescit semper per vim ipsi contrariam & aequalem, qua descensus corporis impediri potest."

That is: *An example* (of "quantity of centripetal force") *is weight, which is greater in a larger body and less in a smaller body, and in one and the same body is greater near the earth and less out in the heavens. This quantity is the centripetency, or propensity toward a centre, of the whole body, and (so to speak) its weight, and it may always be known from the force opposite and equal to it, which can prevent the body from falling.*

Here we can see how Newton, quite analogous to the above explained relation between the term "quantity of matter" and its name "mass" in definition 1, at first defines *quantitatively* the "vis centripetae quantitas", the *quantity of centripetal force*, and then proposes a *semantic equivalent name* for that quantity, which name is "weight."

But which concept of "mass" does Newton use in this context? His definition of the "vis centripetae quantitas" refers to proportionality between this quantity and the (quantity of) *motion* (generated in a given time). The quantity of motion is defined in Newton's definitio 2 as *the product of a body's velocity in its quantity of matter*. Obviously this "quantity of matter" is the very same thing as it is defined in definitio 1. As little as definitio 1 refers to some "inertial mass", so little does definitio 8 refer to some "gravitational mass". For Newton, there is *just mass*, or *quantity of matter*, which quantity must certainly be one and the same thing in a body no matter if the body falls, or if it moves on a plane. So, Newton's evident distinction between quantity of matter, as defined in definitio 1, and weight, as defined in definitio 8, never meant to distinguish between "mass" and "weight" as two different concepts of mass (gravitational vs. inertial mass). Consequently the general confounding of these notions of "matter" and "force", to which Jammer refers ( COM 2000 p. 90), did not indicate that “an unambiguous terminology to accentuate the distinction was not yet available”, as Jammer puts it. Rather it meant just another effort of the general aim at reducing “force” (weight) to only *a quality, or property of matter* (the later on so-called *passive gravitational mass*). How this idea was effectively brought forward in the course of the 20th century, can be seen in a modern lexicon of physics, which tells us that *“weight” (symbolized by* m !) *means the mass of a body which is determined by weighing* [22].

Max Jammer rightly states on p. 91 (COM 2000) that Henri Poincaré in 1908 was one of the first “to use explicitly a term to denote gravitational mass”. Poincaré, as quoted by Jammer, defined mass “firstly, as the quotient of the force by the acceleration, which is the measure of the body’s inertia, and secondly, as the attraction exercised by the body upon a foreign body…”. What one sees here is Poincaré’s absolute ignorance with respect to Newton’s definitions of “quantitas materiae”, of “vis inertiae”, and of “vis centripetae quantitas motrix”. Poincaré’s definition corroborates what I have stated at the beginning of this paper that “Newton’s authentic theory of motion has nearly nothing to do with classical mechanics”.

Next to Poincaré it was, however, the great Albert Einstein, celebrated conqueror of Newton’s allegedly deficient mechanics, who emphatically asserted that this "Newtonian mechanics" would define inertial mass and gravitational mass differently. And on the background of this assertion (unfounded as it was with respect to true Newtonianism), he established what he sometimes called “the happiest thought in my life” – the idea of an equivalence of inertial and gravitational mass, the later on so-called “equivalence principle of Einstein.”[23] I dare to call it one of the most curious paradoxes of the history of science that Einstein’s highly praised abolition of the arbitrary distinction between different kinds of mass, as we can see now, actually meant a partial return to Newton’s true theory which never had implied any such thing. Insofar as Einstein himself unjustly, as we know, first insisted on attributing to Newtonian mechanics an arbitrary and unexplained distinction of concepts which he afterwards proudly removed, praising this removal as an achievement which should definitely establish his general relativity over Newtonian mechanics[24], the case somehow reminds of a man who returns the purloined puppy to its owner, claiming the finder’s reward. Einstein could have avoided this harsh judgement by carefully studying and respecting the true authentic theory of his great predecessor, instead of establishing his theories on the dogmatic deviations from authentic Newtonianism to be found in the works of e.g. Ernst Mach and Henri Poincaré.

**VI On the origin of "inertial" vs. "gravitational" mass, on Einstein's equivalence principle, and on the Newtonian concept of "force" in general.**

Einstein, when he presented his principle of equivalence, much in the way of Poincaré presupposed, as a definition of (inertial) mass, the formula $\boldsymbol{F} = m\boldsymbol{a}$, or $m = \boldsymbol{F}/\boldsymbol{a}$ [24], which formula stems not from Newton but from Euler, as I have already stated in section I above. It is clear, however, that Einstein as well as all his scientific contemporaries attributed the law that states the equality of "force" and "mass-acceleration" to Newton as a matter of course.

Now, if we look nearer at this foundation of post-Newtonian classical mechanics, we find that it formally allows for two presentations: One (which we may call the "inertial version") reads $\boldsymbol{F}/\boldsymbol{a} = m = constant$, to say that a definite body $m$ will always be accelerated in proportion to the accelerating force $\boldsymbol{F}$, since this force $\boldsymbol{F}$ and the corresponding acceleration $\boldsymbol{a}$ of the body are *proportional* in this case: $\boldsymbol{F}/\boldsymbol{a} = m = constant$ implies by definition of proportionality that $\boldsymbol{F}$ and $\boldsymbol{a}$ are proportional to each other: $\boldsymbol{F} \propto \boldsymbol{a}$ , with $m$ to represent the *constant of proportionality*.

Alternatively, we may, however, as well put $\boldsymbol{F}/m = \boldsymbol{a} = constant$, to say that any accelerating force $\boldsymbol{F}$ will accelerate the motion of any body $m$ always according to the same constant quantity $\boldsymbol{a}$ of acceleration. As this is supposed to be the case with acceleration caused by the gravitational force, we may call this version of the formula $\boldsymbol{F}/m = \boldsymbol{a}$ the "gravitational version". This version of $\boldsymbol{F}/m = \boldsymbol{a} = constant$, in contrast to its "inertial version", implies by definition of proportionality that $\boldsymbol{F}$ and $m$ are proportional to each other: $\boldsymbol{F} \propto m$, with acceleration $\boldsymbol{a}$ serving as *constant of proportionality* in this case.

Now, if one would use the "inertial version" of the formula $\boldsymbol{F}/\boldsymbol{a} = m$ as a definition of $m$, as Henri Poincaré did, one would obtain an "inertial" definition of $m$, that is a definition of some "inertial mass" $m_i$. On the other hand, with the "gravitational version" of that formula, one would obtain a "gravitational" definition of $m$, that is a definition of some "gravitational mass" $m_p$. So we understand that the mathematical groundwork for a distinction between "inertial" and "gravitational" mass was prepared as soon as the formula $\boldsymbol{F} = m\boldsymbol{a}$ appeared on the stage. And this happened for the first time with Leonhard Euler's "Mechanica" of 1736, even though it can be shown that Euler's concept was rooted in Leibniz's distinction between “vis mortua”, the *dead force*, and “vis viva”, the *living force*, published in 1695, and meant as a corrective answer and counter-demonstration to Newton’s *Principia* of 1687 [25].

Returning to Einstein again, we find that he, in the presentation of his equivalence principle, asserted that Newton’s equation of motion through a gravitational field should be written

$$m_i \times \boldsymbol{a} = m_p \times \boldsymbol{I},$$

with $\boldsymbol{I}$ representing the “intensity of the gravitational field”, i.e. the constant acceleration within that field[26]. Obviously Einstein simply *equated* those different two versions of $\boldsymbol{F} = m\boldsymbol{a}$ which we have just developed. It is clear, however, that this operation requires $\boldsymbol{F} = \boldsymbol{F}$ which is indeed given in the special case of only gravitationally accelerated motion, where $\boldsymbol{a} = \boldsymbol{I}$ is also given, and consequently there inevitably results $m_i = m_p$ without any need to apply a “principle of relativity”. This principle, however, Einstein erroneously held to be a necessary part of his pretended demonstration of an *identity of character* (German: “Wesenseinheit”) of two seemingly so different concepts of mass, by means of his so powerful general relativity theory.

In any case, it was the concept of “force” of classical mechanics, which provoked the distinction between two concepts of mass, and Einstein’s miraculous restoration of the “Wesenseinheit” of mass as well. Needless to say it once more that this classical concept of “force” has nothing to do with Newton, and that we meet here with another cogent example of classical mechanics being different from Newtonian mechanics. However, for the sake of completeness an additional word must be said about Newton’s *authentic concept* of “force”, as it is present in his second law of motion. I will concentrate on this second law because it is generally but erroneously held to be an equivalent of the formula $\boldsymbol{F} = m\boldsymbol{a}$.

What does this formula say? It states that some force, $\boldsymbol{F}$, is equal to mass-acceleration, $m\boldsymbol{a}$, i.e. to the accelerated motion of a body, $m$. We are left to conjecture that this should mean: A force, $\boldsymbol{F}$, which acts on a body, $m$, always makes the body move in an accelerated manner, i. e. that “force” is always an *accelerating* agent identical with the measure $m\boldsymbol{a}$ of accelerated motion.

Now, Newton’s authentic second law in its very different central message reads

> “Mutationem motus proportionalem esse vi motrici impressae”, that is:

*The impressed motive force is proportional to the change in motion.*

Quite obviously this statement is not synonymous with its common rendering into $\boldsymbol{F} = m\boldsymbol{a}$, and consequently our common use of "force", $\boldsymbol{F}$, *must differ dimensionally from Newton's force,* $\boldsymbol{F}_{newton}$ (in the following: $\boldsymbol{F}_n$). Even though it has sometimes been noticed that Newton's law is one *not on acceleration*, but on *change of momentum* (Latin "mutatio motus"), i.e. $\boldsymbol{F}_n \propto \Delta(m\boldsymbol{v})$, little attention has been paid so far to the fact that Newton puts the force *not equal* to its effect on a body's motion, but *proportional* to it. Some scientists, who at least happened to meet this expression as something to be explained, thought to escape the problem proposing to render $\boldsymbol{F}_n$ and its proportional counterpart into equals *by a proper choice of units*[27]. This proposal, however, proves circular and invalid, since it implicitly presupposes that the two terms, which Newton put proportional, *should have identical dimensions* – which ultimately would presuppose that they were equals. This ultimate conclusion, moreover, would contradict Newton's clear words, because he explicitly put the terms not equal, but *proportional*, which term plays a dominant role in his decidedly *geometric* treatment of the theory of motion. This Newtonian theory as well as its Galilean foundation could only have been composed as a *mathematical* theory by means of the Euclidean theory of proportion, as a theory of mathematical relations between natural entities *of different kinds*.

The relevance of this argument can clearly be seen in Newton's *Principia*, Book 1, section 1, *Scholium* after *Lemma X* that reads:

"Si quantitates indeterminatae diversorum generum conferantur inter se, & earum aliqua dicatur esse ut est alia quaevis directe vel inverse: sensus est, quod prior augetur vel diminuitur in eadem ratione cum posteriore, vel cum eius reciproca."

That is: *If indeterminate quantities of different kinds are compared with one another and any one of them is said to be directly or inversely as any other, the meaning is that the first one is increased or decreased in the same ratio as the second or as its reciprocal*[28].

As "to be directly or inversely as any other" is tantamount with "to be directly or inversely proportional", the quoted passage contains the germ of Newton's way to use proportion theory, i.e. mainly as the only available method to determine mathematical laws in the respective behavior of *quantities of different kinds* such as "forces" and "motions", or "changes of motions".

What does this mean for the correct interpretation of Newton's second law? It means that the algebraic representation of this law requires making explicit a constant of proportionality as follows:

$$\boldsymbol{F}_n \propto \Delta(m\boldsymbol{v}) \text{ means: } \boldsymbol{F}_n : \Delta(m\boldsymbol{v}) = constant\ .$$

The constant represents the factor of proportionality. As in Euclid's theory every true proportion consists of four links, A : B = C : D (i. e. the "quaternary proportion", or *tetraktys*), we must write Newton's factor of proportionality as a relation, e. g. X : Y . So we obtain

$$\boldsymbol{F}_n : D(m\boldsymbol{v}) = X : Y = constant.$$

The question then is what the meaning behind this yet unknown "Newtonian constant"[29] might be. Since I have already shown all of it elsewhere[30], I can be short here: The Newtonian constant is composed of a discrete element of space, $\Delta s$, and a discrete element of time, $\Delta t$, and this constant quotient, $X : Y = \Delta s/\Delta t$, is geometrically identical with that constant $c$ which came to the fore in Maxwell's theory, later on called "vacuum velocity of light", to play a major role in modern physics, as everybody knows.

For the identity of my "Newtonian constant" $\Delta s/\Delta t$ with the constant vacuum velocity of light, $c$, I refer to a convincing paragraph in Max Jammer's "The Philosophy of Quantum Mechanics" of 1974, which reads:

"The view that a formal identity between mathematical relations betrays the identity of the physical entities involved - a kind of assumption often used in the present-day theory of elementary particles - harmonizes with the spirit of modern physics according to which a physical entity does not do what it does because it is what it is, but is what it is because it does what it does. Since what it 'does' is expressed by the mathematical equations it satisfies, physical entities which satisfy identical formalisms have to be regarded as identical themselves, a result in which the mathematization of physics, started by the Greeks (Plato), has reached its logical conclusion."[31]

## VII Some remarks on" active gravitational mass" $m_a$, from the Newtonian point of view.

In chapter 4 of COM 2000, Max Jammer states that presumably only in the year 1957 the *dichotomy* of inertial and gravitational mass became explicitly extended to a *trichotomy* trough a definite distinction of gravitational mass into "active" and "passive". He refers to Hermann Bondi's "often quoted essay on negative mass in general relativity" [32]. No doubt that Jammer will be right insofar as Bondi was the first *to mathematically define* $m_a$ and $m_p$ differently. From the Newtonian viewpoint, however, one must admit that *the idea* of an active gravitational mass is much older.

To quote Jammer, active gravitational mass is commonly assigned to every body, and it "specifies the body's role as the source of a gravitational field" (COM 2000 p. 90). Obviously the underlying idea is that of an active power of all bodies to attract other bodies to their centre if they enter the range of the gravitational field to surround the attracting body. Active gravitational mass $m_a$, then, is conceived *as yet another quality of matter.* It means that well-known conception of matter *to act at a distance*, e. g. of the earth being able to grasp at an apple and make it fall from the tree to the ground, or to grasp even at the moon and, by combination with a tangential uniform motion of the moon, make it revolve round the earth.

It is this view that commonly is imputed to Newton, and it seems to be a true rendering of his gravitational theory, as it is explained in the *Principia*, definitions 5 to 8. Looked at more closely, however, these definitions do not support it.

Firstly, in contrast to the generally taught theory of matter as being endowed with a quality to act at a distance, we should see that Newton *does not define a quality of matter, but rather a force again,* which force he calls by the name of "vis centripeta", *the centripetal force* (def. 5).

According to this definition 5, the centripetal force is characterized through its ability to make bodies tend to some central point, as gravity makes bodies tend to the centre of the earth, and as the magnetic power makes iron approach a magnet. Definition 6 defines *quantitatively* the absolute quantity of the centripetal force as proportional to the efficacy of that cause which propagates it from the centre through the surrounding regions.

Definitions 7 and 8, then, deal with the *quantities of motive action* of the centripetal force on bodies. According to definition 7, the “accelerative quantity” of this force exerts an accelerating effect on the motion of bodies in proportion to the velocity generated in a given time. The explanation that Newton gives here shows that this accelerating quantity of the centripetal force *depends on the distance between the centre and the locus where it acts on the body*.

Secondly, then, this explanation of Newton, again in contrast to the general belief, makes it clear that the gravitational action at a body is not due to a power of the central body to act on other bodies at a distance, but rather it is an effect of a *local force to act on a body where it is.*

According to this view, one is indeed forced to admit that Newton’s true theory implies a notion of a “gravitational field” to spread around central bodies, and of *local action* on other bodies in this field. Surprisingly or not, it is Max Jammer who must be given the credit of having drawn public attention to this aspect of Newton’s definition 7, in his “Concepts of Force” of 1957 (p.123). Here, referring to the said definition 7, Jammer states that Newton’s words “seem to suggest that Newton already was thinking of force in the conception of field.” However, for the sake of historical correctness, one must point to the fact that already James Clerk Maxwell, in a treatise of 1861, giving an historical account of theories of action at a distance, stated that Newton had strongly rejected interpretations of his theory in the sense of instantaneous action at a distance[33]. Maxwell quotes from Newton’s letter to Bentley of Feb. 25,1692-3 which was then widely unknown, a today well-known phrase which, however, is mostly quoted incompletely and interpreted wrongly. Most authors suppress its introducing words. The complete phrase refers to an argument of Bentley against action of matter at a distance. Newton wholeheartedly agrees with Bentley, and, to underline his view, says:

“It is inconceivable, that inanimate brute matter should, without the mediation of something else, which is not material, operate upon, and affect other matter without mutual contact; as it must do, if gravitation, in the sense of *Epicurus*, be essential and inherent in it. *And this is one reason why I desired you would not ascribe innate gravity to me.* That gravity should be innate, inherent and essential to matter, so that one body may act upon another at a distance through a *vacuum*, without the mediation of anything else, by and which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking, can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers" [34] (my italics).

This view of Newton's gravitational theory as one of (non-instantaneous) local action in a "field" recently has gained much support from Howard Stein's essay on "Newton's Metaphysics". Stein, after having quoted from Newton's explanation to definition 8, says:

"This passage describes the conception of what in a later terminology is called *a field of force*, distributed about – and everywhere tending towards – a center. The 'absolute quantity' of this force (this field) is meant to characterize the strength of the field *as a whole* – the 'efficacy of the cause' by which it is produced, or 'propagated through the spaces round about'; again, in later terms, is the 'source-strength' at the center of the field. The 'accelerative quantity' is *meant* to characterize the intensity of the field *at any given place* (and in the special case of gravitation, the 'acceleration due to gravity' at the place in question successfully does so). Finally, the 'motive quantity' characterizes the action of the field upon an *actual body*; it measures, in other words, the force *impressed upon* a body by the field – the impressed force that has the given (field of) centripetal force as its 'origin'. In the case of gravity, the motive quantity of the force on a body is simply the *weight* of that body" [35].

So, where in history can we find somebody who was the first to assert that bodies could act upon bodies instantaneously, and at an arbitrary distance? As this work should not be seen as a historical one, it should suffice to point to Immanuel Kant again, who, in his "Metaphysische Anfangsgründe" of 1786, explicitly attributed to matter a quality of "actio in distans" [36]. For the sake of completeness, however, one should also have a look at Kant's sources, Leonhard Euler [37], G. W. Leibniz [38], and ultimately René Descartes.

After all, it should be evident from the quoted letter to Bentley that Newton absolutely rejected the then already spreading action-at-a-distance interpretation of his theory because of its materialistic implications. Matter, for Newton, was "inanimate" and "brute", and absolutely passive, i.e. unable to exercise *by itself* any power or force. Consequently, the gravitational force which makes bodies tend to leave their proper places and move to a centre had to be conceived as a *local* action of *something immaterial* which was located in empty space at the very place of a gravitating body. Needless to add that Newton shared the common conviction that *no body and nothing can act where it is not* (as Samuel Clarke expressed the Newtonian view, so far agreeing with Leibniz, in his exchange of papers with G. W. Leibniz of 1715/1716 [39]).

To sum it up, even the notion of active gravitational mass, as well as any other attribution of powers or forces to matter as its active or passive qualities, has nothing to do with Newton’s authentic theory of motion and gravitation. Newton’s theory throughout is one of *local interaction* between “force” and “matter” (at rest or in motion), which observation inevitably implies to understand both these entities as real, but *different* constituents of nature. It is clear, then, that Newton's "mass" *as a definite quantity* cannot be *a variable*. On the other hand, the appearance of "mass" as a variable in Einstein's *special relativity* is evidently linked up to its above criticized "classical" misinterpretation as a *quality* of matter. Should one not infer then that Einstein's theories *at least in some respect* were actually developed on the basis of a corrupted version of Newton's theory of motion, and might, *in some other respect, namely insofar as they work*, mean nothing but a restoration of Newton's authentic theory?

With respect to "force" we have already seen above that the force that Newton presents in his second law has to be distinguished from motion, or change of motion, through its different dimensions, a difference that requires proportion theory as the only proper tool to establish meaningful mathematical relations between “force” and “motion”. Consequently, Newton’s “force” $\boldsymbol{F}_n$ *proportional to* "change of motion" $\Delta(m\boldsymbol{v})$ must substantially differ from its "classical" representation, and it must mean something immaterial, even though it represents a real constituent of nature. This immateriality of Newton's "force" is indicated in Newton’s above quoted letter to Bentley, and was also clearly seen from a different point of view e. g. by the late Newton scholar Betty Dobbs[40]. To adopt this view gives Newton’s theory back its full powers, as a realist theory of motion in time and space, and of *local non-instantaneous interaction*, say *interaction in space and time*, as a careful rendering of his second law brings to light the relation $\boldsymbol{F}_n/\Delta(m\boldsymbol{v}) = c$, which c means a constant quotient $\Delta s/\Delta t$ of discrete elements $\Delta s$ of quantized space and of discrete elements $\Delta t$ of quantized time.

After all, I want to express the hope to have shown how the dichotomy or even trichotomy of “mass” results only from arbitrarily reducing Newton’s *real forces of nature* to mere *qualities of matter*, say in consequence of an omnipresent predominant *materialist philosophy of science*.

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1) M. Jammer, Concepts of Mass in Classical and Modern Physics (Harvard University Press, Cambridge, Mass., 1961); Concepts of Mass in Contemporary Physics and Philosophy (Princeton University Press, Princeton, New Jersey, 2000). In the present paper, I use the abbreviations COM 1961, COM 2000. Quotes of COM 1961 refer to the German edition, Der Begriff der Masse in der Physik (Wissenschaftliche Buchgesellschaft Darmstadt, Darmstadt, 1981).

2) E. J. Dijksterhuis, De mechanisering van het wereldbeeld (J. M. Meulenhoff, Amsterdam, 1950), quotes the fairy-tale of the Emperor's new clothes in order to characterize the situation that everybody thought to find the $\boldsymbol{F} = m\boldsymbol{a}$ in Newton's second law, whilst that law in fact states something very different.

3) M. Jammer, Concepts of Force (Harvard University Press, Cambridge, Mass., 1957), pp. 124, sees some difference between the classical $\boldsymbol{F} = m\boldsymbol{a}$ and Newton's second law, struggles with the correct interpretation of Newton's law, but fails to notice the most important point, which is the *proportionality* (not: equivalence!) between Newton's motive force and its effect on the state of a body.

4) L. Euler, Découverte d'un nouveau principe de méchanique, Mémories de l'Académie des Sciences de Berlin **6**, 185 (1750).

5) I. Newton, Mathematische Grundlagen der Naturphilosophie, E. Dellian ed.(F. Meiner Verlag, Hamburg, 1988). My introduction to Newton's work contains a condensed line of reasoning for the correct interpretation of the second law.

6) E. g. Newton in the *Principia*, which he composed as a treatise on the motion of bodies, does not define *velocity, uniform motion, and uniformly accelerated motion* which are the main subjects of Galileo's *Discorsi,* 3rd day.

7) Newton's high esteem for classical geometry can be best understood from his "Praefatio ad Lectorem", the preface of May 8, 1686, to the first (1687) edition of his *Philosophiae*

*Naturals Principia Mathematica.* There he praises *geometry* as that part of mechanics which teaches the art of exact measurement, exactly and by demonstration. Actually, in Newton's time geometry as the *art of measuring* was still distinguished from arithmetic and algebra as the *art of calculating*.

8) Cf. e. g. S. Chandrasekhar, Newton's *Principia* for the Common Reader (Clarendon Press, Oxford, 1995), which book the Newton scholar I.B. Cohen criticizes as "an essentially non-historical work", which "disdainfully and cavalierly dismisses the whole corpus of histori-cal Newtonian scholarship"; see I. B. Cohen and A. Whitman, Isaac Newton - The Princi-pia (University of California Press, Berkeley-Los Angeles-London, 1999), p. 295.

9) M. Erdmann, Masse macht's, aber was macht Masse? in H. Müller-Krumbhaar, H.F. Wagner eds., *... und Er würfelt doch* ( Wiley-VCH Verlag Berlin GmbH, Berlin, 2001).

10) E. Mach, Die Mechanik in ihrer Entwicklung, historisch-kritisch dargestellt (Minerva, Frankfurt a. M., 1982), p. 210.

10a) I. Bernard Cohen, Newton's Concepts of Force and Mass, in: The Cambridge Companion to Newton, I. Bernard Cohen and George E. Smith eds. (Cambridge University Press, Cambridge/UK, 2002), p. 59.

11) It is well known that Mach was a dogmatic anti-atomist who ignored all experimental evidence of a quantized structure of matter until to his death in 1916.

12) Cf. E. Cassirer, Leibniz' System in seinen wissenschaftlichen Grundlagen (Georg Olms Verlag, Hildesheim-New York, 1980), part 1 chapter 4 „Das Problem der Kontinuität"; see also I. Kant, Metaphysische Anfangsgründe der Naturwissenschaft, in: Kant's gesammelte Schriften, Königl. Preussische Akademie der Wissenschaften ed., G. Reimer Verlag, Berlin, 1911), Vol. IV p. 536 f. „Mechanik", esp. Lehrsatz 1, Beweis: „Die Materie ist ins Unendliche teilbar, folglich kann keiner ihre Quantität durch eine Menge ihrer Teile unmittelbar bestimmt werden (sic)." Kant here presupposes that matter should be divisible ad infinitum ("ins Unendliche teilbar"), and therefore infers that its quantity could never be determined as some quantity of its parts - in evident contradiction to Newton's definitio 1, and as an expression of Kant's belief in matter to be an unstructured

Cartesian continuum.

13) M. Jammer, COM 1961, 91.

14) Cf. ref. 9).

15) L. Euler, Briefe an eine deutsche Prinzessin über verschiedene Gegenstände aus der Physik und Philosophie, U. Sexl, K. von Meyenn eds. (F. Vieweg & Sohn, Braun-Schweig/Wiesbaden, 1986), p. 143, 124th letter.

16) Cf. ref. 12 (I. Kant).

17) S. Clarke, Der Briefwechsel mit G. W. Leibniz von 1715/1716, E. Dellian ed. (F. Meiner Verlag, Hamburg, 1990) p. 100 (Leibniz' fifth letter, nr. 113).

18) I. Kant, p. 549/550 (3. Hauptstück Mechanik, Lehrsatz 4 Anmerkung 1 und 2: „Die Benennung der Trägheitskraft (*vis inertiae*) muss also unerachtet des berühmten Namens ihres Urhebers aus der Naturwissenschaft gänzlich weggeschafft werden.....“). Cf. M. Jammer, COM 1961, p. 87/88: “Schließlich verwirft Kant in seinem Werk “Die metaphysischen Anfangsgründe der Naturwissenschaften“, das 5 Jahre nach seiner „Kritik der reinen Vernunft“ veröffentlicht wurde, Newtons Begriff der *vis inertiae* völlig...“.

19) E. Mach p. 240, referring to Newton's second law (which he erroneously interprets as a law of *accelerating* force), criticizes it as tautologic, stating that the “forces” were already defined in Newton's definitions. And these definitions – he continues – *should correspond to qualities of bodies given from experience (*“… dass die Definitionen … in der Erfahrung gegebenen *Eigenschaften der Körper* entsprechen” – my italics).

20) M. Jammer, COM 2000, p. 8.

21) Cf. M. Born, Die Relativitätstheorie Einsteins (Springer Verlag, Berlin, 1969), p. 37: „Häufig spricht man das Gesetz von der Proportionalität des Gewichts mit der Masse so aus: *schwere und träge Masse sind gleich*....Dass dieses Gesetz sehr genau gilt,

hat bereits Newton gewusst...“. That is*: The proportionality of weight and mass is often expressed by saying that ‘heavy mass and inert mass are equal’. The very exact validity of this law was already known to Newton….*

22) P. Waloschek, Wörterbuch der Physik (Deutscher Taschenbuch Verlag, München, 1998) p. 176 („Gewicht“).

23) M. Born p. 271.

24) A. Einstein, Grundzüge der Relativitätstheorie (F. Vieweg & Sohn, Braunschweig, 1969), p. 20, 23, 50, 55.

25) G. W. Leibniz, Specimen Dynamicum, H.G. Dosch, G. W. Most, E. Rudolph eds.(F Meiner Verlag, Hamburg, 1982).

26) A. Einstein p. 59/60.

27) J. Mittelstrass, Neuzeit und Aufklärung (W. de Gruyter, Berlin/New York, 1970) p. 288; R. Torretti, The Philosophy of Physics (Cambridge University Press, Cambridge, 1999), p. 47.

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28) I quote the English version from Isaac Newton, The Principia, I. B. Cohen, A. Whitman eds. (University of California Press, Berkeley-Los Angeles-London, 1999), p. 438.

29) E. Dellian, Philos. Nat. **22**, 400 (1985). Here for the first time the interpretation of Newton's second law to uncover a "Newtonian constant" was published.

30) E. Dellian, Inertia, The Innate Force of Matter: A Legacy from Newton to Modern Physics, in: P. B. Scheurer and G. Debrock (eds.), Newton’s Scientific and Philosophical Legacy (Kluwer, Dordrecht, 1999) p. 227; Physics Essays **3**, 365 (1990), Philos. Nat. **36**, 27 (1999).

31) M. Jammer, The Philosophy of Quantum Mechanics (J. Wiley & Sons, New York etc.,

1974), p. 50.

32) H. Bondi, Negative Mass in General Relativity, Rev. of Modern Physics **29**, 423 (1957).
33) J. C. Maxwell, Scientific Papers (The University Press, Cambridge, 1890), Vol. 1, p. 451, 454.

34) I quote the Latin from I. Newton, Opera quae exstant omnia, S. Horsley ed. (J. Nichols, London, 1779-1785).

35) H. Stein, Newton's Metaphysics, in: The Cambridge Companion to Newton, I. B. Cohen and G. E. Smith eds. (The University Press, Cambridge, 2002), p. 286.

36) I. Kant, p. 511 (Dynamik, Erklärung 6).

37) L. Euler, cf. ref. 15), p. 83-85.

38) G. W. Leibniz, cf. ref. 25), p. 7/8, where he attributes „active force" to matter as one of its qualities.
39) S. Clarke, p. LXVI, 145.

40) B. J. T. Dobbs, The Foundations of Newton's Alchemy (The University Press, Cambridge, 1975), p. XII, 210; also: Newton's Alchemy and his 'Active Principle' of Gravitation, in: Newton's Scientific and Philosophical Legacy, P. B. Scheurer and G. Debrock eds. (Kluwer, Dordrecht, 1988), p. 55.