The Newtonian Constant

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by Ed Dellian.

I

Isaac Newton’s *Principia* are turning 300.[1] They are hardly ever read, except by historians of science, who tend to confine themselves to historical aspects and, as far as physics goes, to identifying the principles of classical mechanics within Newton's opus summum, despite the fact that its precise wording yields little in this regard.[2] The upcoming anniversary is an inviting occasion to attempt a reconstruction of the real physical contents of Newton’s principles. This causes a previously unknown "Newtonian constant" to emerge, which should certainly interest philosophers as well as historians of science and physicists. This interest may suffice to justify presenting the attempt and its results here. Historical details are thus kept to a necessary minimum.

II

The point of approach is Newton’s Second Axiom or second law of motion. It states: "Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur." *A change in motion is proportional to the motive force impressed; and takes place along the straight line in which that force is impressed.*

It has long become customary to regard this as defining force as being equal to the time derivative of change in momentum, or in simpler terms, to interpret: *force equals mass times acceleration*. However, among historians of science there is a growing awareness that Newton’s law doesn’t speak of a time derivative of the change in momentum.[3]

It is well known that Sir Isaac repeatedly reworked his formulations and chose his words meticulously. So it cannot be permissible to just read the time derivative into the Second Axiom simply because "...axioms were not Newton’s strength"[4]. The Principia stipulates clearly in many places that Newton’s elementary concept of impressed force is proportional to the

[1] Philosophiae naturalis,principia mathematica, London 1687; English translation; Andrew Motte, ed. 1846. Newton’s Foreword to the 1st edition bears the date: Cambridge, May 8, 1686.

[2] E.J. Dijksterhuis, Die Mechanisierung des Weltbildes, Springer Berlin-Heidelberg-New York, 1983 (p.528).

[3] Max Jammer, Concepts of Force, Harvard University Press Cambridge/Mass., 1957 (p.124); Brian D. Ellis, Newton's Concept of Motive Force, Journ.Hist.Ideas (23) 1962 (p. 273 ff.); I. Bernard Cohen, The Newtonian Revolution, Cambridge University Press, 1980 (p. 172 ff.); Werner Kutschmann, Die Newtonsche Kraft, Studia Leibnitiana, Sonderheft 12, Steiner, Wiesbaden, 1983.

[4] E.J. Dijksterhuis ibid.

change in momentum only (no time derivative), and so is correctly expressed in the formula $F \propto \Delta(mv)$[5].

Written as an equation, this formula requires a proportionality factor:

$$F = \Delta(mv) \times c \; ; \qquad (1)$$

c is the *Newtonian Constant*.

Where this constant appears in elementary representations of Newton's principles of mechanics it is rapidly suppressed. The authors see fit, by choosing appropriate units, to equate it with 1 and so remove it from the definition of force.[6]

Now it is plain to see that this is only possible if c is *without dimension*, or, put differently, if F and $\Delta(mv)$ in eq. (1) are *dimensionally equivalent*. This leads to a philosophical problem, since force F and change in motion $\Delta(mv)$ are related to one another in Newtonian experimental natural philosophy as *cause and effect*. This means however that "force" and "motion" (change in motion) must stand for *heterogeneous physical entities*. The cause "force" and the effect "change in motion" must possess, expressed in modern terms, *different dimensions.* It follows that their quotient, the proportionality constant c in eq. (1), bears also proper dimensions.

For that reason, this *Newtonian Constant* cannot be removed from the definition of force, not even by equating it to 1, without altering its physical contents.

III

So if eq. (1) is a correct rendering of Newton's *Second Axiom* we can assume that the fundamental law of classical mechanics, the formula "force equals mass times acceleration", diverges from this axiom by virtue perhaps of being developed out of Newton's doctrine by others. Certain studies in the history of science seem to affirm this assumption.[7] At the same time, historical considerations make it necessary to emphasise that the essential characteristic of this formula, namely the equivalence of force F with its effect, is a product exclusively of the mind and philosophy of G.W. Leibniz. During his sojourn in Paris from 1672-1676, Leibniz's engagement with occasionalistic thinking led him to forward the proposition of the equality of cause and effect; *causa aequat effectum*.[8] Until then, the forces and their

[5] Cf. Newton‘s explanations of his 2nd Axiom, and Scholium after Corol. VI to the axioms.

[6] Jürgen Mittelstrass, Neuzeit und Aufklärung, de Gruyter Berlin-New York, 1970 (p.288). Steven Weinberg, Teile des Unteilbaren, Spektrum der Wissenschaft, Heidelberg 1984 (p.139); cf. also Brockhaus Enzyklopädie 1970 unter "Kraft".

[7] Thomas L. Hankins, The Reception of Newton's Second Law of Motion in the Eighteenth Century, Arch.Int.Hist.Sci. (20) 1967, p.43-65.

[8] H.-J. Hess, Die unveröffentlichten Naturwissenschaftlichen und technischen Arbeiten von Leibniz, Studia Leibnitiana Supplementa Bd. 17, Steiner Wiesbd. 1978 (p. 183 ff, 202 ff.).

effects had been viewed within mechanics not only as distinct but even as incommensurable entities.[9] Leibniz proceeds to declare the as yet unsubstantiated claim *causa aequat effectum* his "first mechanical axiom".[10]

On this basis it was however now possible to formulate an analytical definition of force (without Newtonian Constant) and to found an analytical mechanics on top of it, as performed to perfection by the Leibniz admirers and followers L. Euler und J.L. Lagrange.[11] In point of fact, what is referred to as classical mechanics and attributed to Newton is essentially the work of these men inspired by Leibniz. The definition of force by the time derivative of change in momentum can be easily brought into agreement with Leibnizian principles, whereas this does not succeed with Newton's *Second Axiom*. Since Leibniz derives his living force $[mL^2/T^2]$ as the product of weight or dead force and distance,[12] there results for this *dead force* or precisely this *weight* the quantity *living force divided by distance* with the resulting dimension $[mL/T^2]$; and this is obviously identical with the analytical definition of force or the expression *force equals mass times acceleration*.[13]

IV

If we stick with the Newtonian equation (1), instead of equating cause and effect as Leibniz does, the question arises of the dimension of the proportionality constant *c*. This attempt to reconstruct Newton's elementary concept of force ushers in the most interesting phase.

We may assume hypothetically that Newton's system of mathematical-philosophical concepts, which he presents in the beginning of the *Principia* in eight *definitions* and three *axioms*, is not self contradictory. This must be verifiable by means of dimensional analysis. The difficulty here is that the contents of Newton's terms and therefore their dimensions are only partially beyond doubt. Nevertheless, the dimensions of the unclear terms can be derived from various proportionality relations, which Newton introduces at the beginning of the *Principia.*

The result is that the Newtonian proportions can only be properly resolved and his terms interpreted without contradiction when in the definition of force *a constant with the dimension* [L/T] *is taken into account*. A geometrical examination of eq. (1) confirms that the Newtonian constant must bear the dimension [L/T]. Furthermore, the same constant can already

[9] John Wallis, Mechanica Sive de Motu Tractatus Geometricus, London 1670 (Proposition VII and Scholium).

[10] H.-J. Hess ibid.

[11] J.L. Lagrange, Mécanique analytique, Paris 1788.

[12] G.W. Leibniz, Brevis demonstratio erroris memorabilis Cartesii et aliorum, Acta Eruditorum März 1686.

[13] Equally Max Planck, Das Prinzip der Erhaltung der Energie, Leipzig 1887 (p. 7); cf. also Richard S. Westfall, Force in Newton's Physics, American Elsevier, New York, 1971 (p. 298).

be seen to occur - as the elementary relation between the element of space and the element of time - in Galilean mechanics.[14]

With this constant, Galilean-Newtonian experimental natural philosophy moves into an as yet unseen proximity to Albert Einstein's special theory of relativity. The close structural affinity between eq. (1) and Einstein's $E = mc^2$ is immediately apparent.

V

As a pure mathematical algorithm, the definition of force in analytical mechanics: "force equals mass times acceleration", is merely a convention, not a physical formula relating to reality. Mathematical formalism first graduates to physical theory when connected with constants external to logic[15], i.e. with natural constants, which establish the theory's relation to reality. This is why the Newtonian eq. (1) is a *physical law*, a *law of nature*, as opposed to the analytical definition of force. It is an explicit version of the *law of causality*, which provides the rule by which mechanical effects (changes in motion) follow from their causes.[16] Force thus retains throughout the ontological status that Newtonian philosophy undoubtedly attributes to it and which it lost when Leibnizian analytical mechanics equated cause and effect. Seen clearly, this loss also leads to the abdication of the law of causality.

This Leibnizian foundation of mechanics proved deficient around the turn of the century. Its revision by Albert Einstein was confined to adapting the mathematical formalism to seemingly new realities. The resulting intellectual situation is frequently regarded as unsatisfying.[17] There is reason for supposing that a physics resting on Newton's fundamental equation $F = \Delta(mv) \times c$ (as introduced above) with a concept of force already containing the constant c (as a characteristic of the theory of relativity) could more convincingly cope with relativistic phenomena than would be possible with a merely mathematical correction of analytical mechanics. It should also be noted that eq. (1) does not exhibit the well known three deficiencies in the analytical foundation of mechanics - the notion of the continuum , instananeous action at a distance, and time reversibility - but rather suggests quantization of the phenomena (since according to Newton the factor m is simply an integer multiplicator), local action (with finite wave-propagation velocity c), and arrow of time (the effect follows its cause at the velocity c), as modern physics requires.

[14] Galileo Galilei, Discorsi, Leyden 1638; German (Unterredungen und mathematische Demonstrationen ...) by Arthur von Oettingen, Ostwalds Klassiker der exakten Wissenschaften Nr. 24 (Leipzig 1904), 3. Tag, Theorem II Proposition II, Zusatz I (EC:AC = NG:CJ = RQ: JO = konstant [L/T].

[15] Max Jammer, Zu den philosophischen Konsequenzen der neuen Physik, in: Voraussetzungen und Grenzen der Wissenschaft (Gerald Radnitzky und Gunnar Anderson Hrsg.), Mohr Tübingen 1981 (p. 136).

[16] Immanuel Kant, Kritik der reinen Vernunft, Riga 1781, zweite Auflage 1786: „Alles, was geschieht (anhebt zu sein), setzt etwas voraus, worauf es nach einer Regel folgt".

[17] Max Jammer ibid. (p. 129).

The problems of formulating a united theory for this modern physics may rest on the impossibility of grasping nature with this kind of one-sided (Leibnizian) rationalism which continues to supply the foundation for theoretical physics. What is needed is instead the rediscovery of the true physical principles of Galilean-Newtonian experimental natural philosophy based on mathematics and experience.

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