# THE REALM OF QUANTUM MECHANICS IN A NUTSHELL

Awaked from Bad Dreams by Means of Euclidean Geometry*

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## 1. What is the general aim of mechanics?

1.1. Mechanics aims at measuring the quantities of phenomenal motions of bodies, and the quantities of the motion-generating non-phenomenal *causes*, the „forces“ or „energies“. Quantum mechanics (QM) in particular, following the findings of *Max Planck* (1900[1]) and *Albert Einstein* (1905[2]), understands the microstructure of these quantities of generating causes and generated effects *as quantized*. Consequently, QM also aims at representing these quantities mathematically as integer multiples of elementary microphysical „quanta“.

1.2. According to *Sir Isaac Newton*, the whole task of mechanics „seems to be to investigate the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces“[3]. Consequently, as *Werner Heisenberg* emphasized, QM as well as science in general tries to understand the *causal* relations between *effects* (generated motions or generated *changes* of motions) and their *causes* (generating forces or energies)[4].

## 2. The basic quantum mechanical quantities of motion and energy.

2.1. The mathematical definition and measure of the quantity of motion $p$ of a body $m$ is given through the product $mv$ of this body $m$ and its velocity $v$ since the time of *Isaac Newton*[5]. The *quantum mechanical* (QM) quantity of motion or „momentum“ $p$ consequently is also defined

$$p = mv . \qquad (1)$$

2.2. The mathematical representation and measure of the quantity of energy $E$ to generate the momentum $p$ of an elementary particle (e.g. a light quantum or „photon“) is given through the product $p \times c$ of the generated momentum $p$ with a constant $c$ [6]. Consequently we obtain the equation

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*Cf. *William Shakespeare*, Hamlet, II.2.250. *Hamlet*: „O God! I could be bounded in a nutshell, and count myself a king of infinite space, were it not that I have bad dreams.“

$$E = p \times c = (mv) \times c \,. \qquad (2)$$

This eq. (2) obviously *not only defines* mathematically the energy *E*, but also reveals the mathematical relation of *E* as *cause* to *p* as its *effect.* Since eq. (2) shows that the ratio of *E* to *p* results in a constant *c*, *E/p = c = constant*, we find *E* and *p* (cause and effect) *proportional* to each other, with the factor *c* denoting the *constant of proportionality*[7)]. Making use of the symbol ∝ for „proportional“[8)] we may write

$$E \propto p \qquad (3)$$

*to represent the principle of QM causality, and the QM law of cause and effect* [9)].

2.3. A second measure of momentum *p* is basic in QM. It is derived from the theory of radiation (the wave theory of heat, light, etc.). This measure is given as a quotient of *Planck's constant h,* and the wavelength $l$ of radiation waves[10)] *:*

$$p = h/l \,. \qquad (4)$$

2.4. A second measure of energy *E* is also basic in QM. It is derived from radiation theory too. In the year 1900, *Max Planck* developed what today is mostly written

$$E = h \times \nu \,. \qquad (5)$$

According to this eq. (5), the energy *E* of radiation is proportional to the observable radiation frequency $n$ ; *E/n = h = constant. Planck's* constant *h* represents the constant of proportionality[11)].

By solving eq. (4) for *h* we obtain $h = p \times l$. Substituting this result for *h* in eq. (5) we obtain

$$E = p \times l \times n \,. \qquad (5a)$$

Now, according to wave theory the product ($l \times n$) always yields the constant quantity *c* to denote the „phase velocity“ of a wave phenomenon: $l \times n = c =$ *constant.* So we may also write eq. (5a)

$$E = p \times c \,, \qquad (5b)$$

i.e. again as *E/p = c,* or $E \propto p$, i.e. a *proportionality* between energy *E* as „cause“, and momentum *p* as its proportional „effect“. As this result harmonizes with eq. (2) and proportion (3), the whole above consideration proves mathematically consistent.

2.5. Finally, however, a third measure of energy $E$ must be considered which is also basic in QM. This measure of $E$ is given through a relation of $E$ and $p$ that *differs* from eqs. (2) and (5b). It is the measure

$$E = mv^2/2 \qquad (6)$$

known from classical mechanics as „kinetic energy". In QM, eq. (6) is mostly transformed by means of eq. (1): In substituting $mv$ by $p$, we obtain as an equivalent of eq. (6)

$$E = p^2/2m\,. \qquad (6a)$$

Starting from this eq. (6a), one consequently obtains for the momentum $p$ the measure

$$p = \sqrt{2mE} \qquad (6b)$$

evidently different from the $p = E/c$ of eqs. (2) and (5b). What is the difference? In eqs. (2) and (5b), the momentum $p$ appears *proportional* to $E$ which proportionality means a *linear* relat-ion of $E$ and $p$. On the contrary, in eqs. (6, 6a, 6b) we find a *squared E-p* relation. I shall ex-plain in paragraph 4 why this squared relation is responsible for the *deterministic* appearance of *Erwin Schrödinger's* wave mechanics.

## 3. *Heisenberg's* Quantum Mechanics and Proportion Theory

In the following, I shall concentrate on the formula (1) - (5b) all of which show or imply a *linear E-p* relation, i.e. a *proportionality* between energy $E$ and momentum $p$, regulated by the constant of proportionality $c$. The Euclidean theory of geometric proportions then for the first time answers some questions which arise from the hitherto *enigmatic structure* of QM[12)].

3.1. The Heisenberg indeterminacy relations derived.

3.1.1.Starting on $E = p \times c$, and replacing the constant $c$ (which, according to its dimensions [L/T], represents a constant velocity - i.e. a constant quotient of elements of space $\Delta l$ and time $\Delta t$ - ) by the quotient $\Delta l/\Delta t$, we obtain $E = p \times \Delta l/\Delta t$, and its equivalent $E \times \Delta t = \mathrm{p} \times \Delta l$ . The dimensions of the products are [$\mathrm{mL^2/T}$], that is the dimensions of *Planck's* constant $h$. Consequently these products should represent $h$, or a multiple of $h$, so that we have obtained

$$1)\ \mathrm{n}(E \times \Delta t) = \mathrm{n}h\,; \quad 2)\ \mathrm{n}(p \times \Delta l) = \mathrm{n}h\ (\mathrm{n} = 1,2,3\ ...), \qquad (7)$$

i.e. an equivalent of the Heisenberg „indeterminacy relations" $\Delta E \times \Delta t \geq h$; $\Delta p \times \Delta l \geq h$. And conversely: If we start on the proportions $E : n$ (*Planck*) $= h = constant = p : 1/l$ (*de Broglie*) which represent an equivalent of *Heisenberg's* relations $\Delta E \times \Delta t = \mathrm{n}h = \Delta p \times \Delta l$ too, since $\Delta E$

$\times\ \Delta t = E : n =$ n$h$, and $\Delta p \times \Delta l = p : 1/l =$ n$h$, we obtain a constant $E$-over-$p$ relation, i.e. a proportionality $E/p = ln = c = constant$ again, in agreement with the result shown in paragraph 2.4.

It must be stressed that the above shown arrangement of the $\Delta p$, $\Delta l$, and the $\Delta E$, $\Delta t$ -relations in an *equation of products* $\Delta E \times \Delta t = \Delta p \times \Delta l$ of course holds also true if these relations are interpreted with respect to $h$ as *inequalities* ($\geq h$) in the usual QM manner, since, on the basis of our starting point $E/p = \Delta l/\Delta t = constant$, the respective products, $\Delta p$, $\Delta l$, and $\Delta E$, $\Delta t$, must necessarily yield *always identical results* $\geq h$, so that $\Delta E \times \Delta t =$ n$h = \Delta p \times \Delta l$ is always valid.

By the way one should note that *Heisenberg's* „indeterminacy" interpretation refers *not to the true measures of the physical quantities involved,* but rather to the *limits of their exact observation,* since in microphysics every act of observation and measurement inevitably interfers with the quantities to be measured, and thus affects the results. This indeterminacy, then, means not a property of nature, but rather an expression for the technical limits of exact measurement in applied microphysics[13)]. So our physically meaningful eqs. (2; 5b) that lie behind our mathematically exact eqs. (7) will refer to *what really happens* (even though not exactly to be observed) between energy and momentum (cause and effect) in micro- as well as in macrophysics [14)].

3.1.2. Now, looking again at eqs. (7), and applying proportion theory: Should we not see these eqs. to represent an *inversely proportional relationship* between the $\Delta E$ and $\Delta t$, and the $\Delta p$ and $\Delta l$ respectively? And should this fact not point to another, namely an *intrinsic* indeterminacy of QM, due to the characteristic of inverse proportionality to represent quantities at inverse magnitudes, so that e.g. to the „sharpest" measurable value of $p$ there inevitably should corres-pond the *„most unsharp"* measurable value of $l$ ? Corroboration of the wide-spread assump-tion that in QM one cannot at the same time measure sharp values of the $E$ and the $t$ („energy" and „time"), or the $p$ and the $l$ („momentum" and „place") respectively, of a moving particle? Which seemingly strange behavior means one of the bad dreams of current QM? The answer is strictly No. As *Peter Kirschenmann* has shown with respect to the Heisenberg relations (cf. reference 12), there is no general inverse proportionality to be seen in these relations because the respective products generally result not in a constant $h$, but in a variable n$h$ (with the exception of the borderline case n = 1). Moreover, by means of *sound* proportion theory we will see again in paragraph 3.3. that the Heisenberg relations represent only *a physical meaningless mathematical intermediary* of the theory, while the basically underlying true and physically meaningful law of motion (the law of *cause* and its proportional *effect*) reads $E/p = c = constant$, implying no indeterminacy at all.

3.2. Why the quantum mechanical operators $\Delta E$, $\Delta t$, and $\Delta p$, $\Delta l$ do not commute.

Normally all the factors of any product, e.g. a × b, can at will be reversed. They are said to „commute“, so that a × b = b × a. The QM operators $\Delta E$, $\Delta t$, and $\Delta p$, $\Delta l$, however, *do not commute*. To alter their order of multiplication means to affect the proper result. This again enigmatic behavior of QM to represent (according to *P.A.M. Dirac*) *the main point of Heisenberg's discovery*: where does it come from? If we make use of our eqs.(7), putting again $\Delta E \times \Delta t$ equal to $\Delta p \times \Delta l$ as in paragraph 3.1.1. (n cancelled out), we will obtain an *equation of products* which, according to proportion theory, can be rearranged into the following quaternary proportion: $\Delta E : \Delta p = \Delta l : \Delta t = constant$ [15)]. Now, since the constant $\Delta l/\Delta t$ equals $c$ (as has been shown in paragraph 3.1.1.), our result again represents a proportionality

$$\Delta E/\Delta p = c \; ; \; \Delta E = \Delta p \times c; \; \Delta E \propto \Delta p \quad , \qquad (8)$$

and these formulae are equivalent to eqs. (2), (3), and (5b) above. So, if we would alter the order of factors of the underlying equation of products, we would obtain not this required proportion $\Delta E/\Delta p = c$, but rather a different one. Consequently we may state: The QM operators $\Delta E$, $\Delta t$ and $\Delta p$, $\Delta l$ do not commute because QM claims a definite proportionality of the $\Delta E$ and the $\Delta p$ (of energy and momentum, of cause and effect) *which condition requires a definite multiplication order* if these operators are arranged in *equations of products* such as the Heisenberg relations.

3.3. What is the meaning of *Planck's* constant $h$?

The transformation of the coupled Heisenberg relations into an $E$-over-$p$ proportionality as shown in paragraph 3.1.1. *eliminates* the constant $h$. A similar result has been obtained in paragraph 2.4, when we from eqs. (4) and (5) derived the energy-momentum proportion of eq. (5b), which is regulated not by the constant $h$, but rather by the constant of proportionality $c$. *Planck's* constant $h$ turns out to represent *only a mathematical intermediary* regulating the basic equation of products that corresponds to the energy-momentum proportionality $E/p = c$. As a matter of fact, every equation of products A×B = C×D that is related to a quaternary proportion A:**C** = D:B = *constant*, as shown in footnote 15), must basically (with n = 1) result in a constant, so that generally n(A×B) = n × (constant) = n(C×D) is valid. And this intermediary constant is not the same as the one to regulate the corresponding quaternary proportion, which can be seen from a simple numerical example. Let for instance 5 times 20 be equal to 2 times 50, then the „constant" is 100. However, the corresponding proportion 5:2 = 50:20 yields a „constant" 2,5. The same result can be obtained if we generalize *Planck's* $E : n = h$ to $a/x = h$, and *de Broglie's* $p : 1/l = h$ to $b/y = h$. Now, according to proportion theory two quantities *a,b,* are *proportional to each other* if $a$ is to another quantity $x$ as $b$ is to another quantity $y$: $a : x = h = b : y$ (this is the quaternary proportion). The constant of proportionality to result from $a/b$, however, is found not $h$, but $x/y$. Consequently *Planck's E* and *de Broglie's p* must be proportional to each other, and the constant of proportionality, i.e. the quotient $E/p$, is found not $h$, but $(n : 1/l) = c$. Q.e.d.

## 4. *Schrödinger's* wave mechanics

### 4.1. What is the aim of *Erwin Schrödinger's* theory?

*Erwin Schrödinger* in the year 1926 developed a new mathematical formalism for QM, starting from the assumption that the QM theory of motion should be represented by a method similar to classical wave theory. This idea was near at hand due to the observation that elementary particles under certain conditions appear in a wavelike fashion. *Schrödinger's* result is nowa-days called „wave mechanics", and the „Schrödinger equation" represents its basic algorithm[16)].

### 4.2. On energy and momentum in wave mechanics.

4.2.1. *Schrödinger's* wave mechanics relates the momentum $p = mv$, and the energy $E$ of a particle moving with velocity $v$, to the characteristics *frequency* $n$ , and *wavelength* $l$ of classical wave theory, according to eqs. (4) and (5). These equations of wave theory happen to yield an energy-momentum proportionality resulting in the wave's always constant *phase velocity* $c$, as has already been shown in paragraph 2.4. *Schrödinger's* theory, however, by using for $E$ the term $p^2/2m$ (cf. eq.(6a)), obtains an $E$-over-$p$ term $E/p = p^2/2m/p = p/2m$ which, with eq.(1) $p = mv$, results in a *variable* phase velocity

$$E/p = v/2\ , \qquad (9)$$

the so-called „phase velocity of the Schrödinger wave“. This result is due to the fact that *Schrödinger's* approach simply makes use of only the classical „squared“ concept of kinetic energy $E = mv^2/2 = p^2/2m$ instead of the QM „linear“ concepts $E = pc = hn$ (eqs. (2) and (5)) as a foundation of QM. This can easily be seen if we, e.g. in eq. (6a), replace the leftside $E$ by $p \times c$, and this phase velocity $c$ by $v/2$ (i.e. the phase velocity of the Schrödinger wave):

$$p \times v/2 \ = \ p^2/2m\ . \qquad (10)$$

In replacing $p$ by $(mv)$ according to eq. (1), we obtain

$$mv \times v/2 \ = m^2v^2/2m, \qquad (10a)$$

or

$$mv^2/2 \ = \ mv^2/2 \qquad (10b)$$

which result demonstrates the background of *Schrödinger's* wave mechanics. Of course this foundation of QM *exclusively on the classical concept of kinetic energy* yields a working formalism for the theory of motion as consistent as e.g. the classical Hamiltonian which is also based on the „squared“ concept $E = mv^2/2$ only.

4.2.2. Textbook versions to infer the Schrödinger equation sometimes present as starting point an identification of the „linear“ and the „squared“ energy terms as introduced in eqs. (2), (5b) on the one hand, and in eqs. (6), (6a) on the other: $p \times c \ = \ p^2/2m$[17]. Once again proportion theory helps us to a deeper understanding of this equation: If we rearrange it to the equation of products $p \times c = p \times p/2m$, by cancelling $p$ on both sides we obtain the absurd result that the *constant* $c$ should be equal to the *variable* $p/2m$. As a matter of fact, since this textbook approach means to equate unequals, its realization inevitably must require demanding mathematical operations which e.g. produce the sophisticated appearance of the Schrödinger equation

$$(\mathrm{i}h) \times [¶y(x,t)/¶t] \ = \ - (h^2/2m) \times [¶^2y(x,t)/¶x^2] \ ^{18)} \qquad (11)$$

This formula, however, only presents the result of the venture to achieve an equivalence of the unequal energy terms $pc$; $p^2/2m$, by means of mathematical operations. Ultimately it amounts to the multiplication of *different* factors ($\mathrm{i}h$), and $-(h^2/2m)$ on the respective sides of the equation of the unequals, in order to make the equation consistent[19] . Since the approach *physically* implies to represent the $E$-over-$p$ relation (the *phase velocity*) not as a constant $c$, but rather as a variable $v/2$, it amounts inevitably to a foundation of QM on the classical concept of kinetic energy only, as has already been shown in paragraph 4.2.1.[20)].

4.2.3. The application of proportion theory to the $E/p = v/2$ (eq.9) that lies behind *Schrödinger's* theory, with $v = l/t$ [dimensions „space L over time T"] allows for the quaternary proportion

$$E : p = l : t \tag{12}$$

(I have dropped the physically meaningless factor 1/2). The corresponding equation of products then reads $E \times t = p \times l = h = constant$. Obviously we have derived a relation very similar to the Heisenberg indeterminacy relations as derived in paragraph 3.1.1. There is, however, a considerable difference to be noticed: In contrast to what I have shown in paragraph 3.1.2, *here we have indeed obtained a strict inverse proportionality* of the *variable* terms *E*, *t*, and *p*, *l* respectively. In the Heisenberg case, the factors $\Delta t$ and $\Delta l$ have represented *constant* elements $\Delta l$ of space [L] and $\Delta t$ of time [T], the products of which *constants* with the *variable* terms *E* and *p* could not produce a *constant h*, rather some variable quantities n*h*, since the product of a constant and a variable always results in a variable. At this point, we must certainly conclude that we, by means of proportion theory, have uncovered a considerable main difference between *Heisenberg's* and *Schrödinger's* mathematical representations of QM. Our finding corroborates our basic thesis on the incompatibility (inequality, incommensur-ability) of different definitions of „energy" behind these theories, and defeats the general belief that both theories were equivalent, as *Erwin Schrödinger* seemingly had proved in 1926[21)]. As I see things, this proof was simply based on non-observance of the difference between the „linear" *E*-over-*p* relation ($E/p = c = constant$) behind *Heisenberg's* theory, and the „squared" $E/p = v/2 = p/2m$, *or* $E = p^2/2m$ which *Schrödinger's* theory is established on.

4.3. Why does *Schrödinger's* theory appear as a *deterministic* foundation of QM ?

QM is often said to reveal an *indeterministic* aspect of nature, while classical mechanics, due to its Hamiltonian foundation on differential equations of motion, appears as a strictly *deterministic* tool. As a matter of fact, indeterminism is certainly a characteristic of the energy-momentum proportionality $E/p = c$, i.e. the natural law of *cause and effect*. *Schrödinger's* wave mechanics, however, shows as *deterministic* as the classical Hamiltonian. Small wonder, though, if one only recognizes as *its main characteristics* the elimination of the genuine „linear“ and true *causal* QM proportionality between energy and momentum, in favour of the classical „squared“ energy concept, as demonstrated in paragraph 4.2, and its mathematical representation *in again differential terms* (i.e. the Schrödinger equation eq. (11))[22].

4.4. On „non-locality“.

The application of *Schrödinger's* theory in microphysics under certain conditions seemingly, as another haunting nightmare, makes one and the same particle appear at different places at one and the same time. This enigma is an endlessly discussed main peculiarity of QM. It can, however, easily be shown that it only results from an intrinsic property of the underlying „squared“ energy term $E = mv^2/2$ (eq. (6)). This concept rests on the assumption that $E$ were proportional to the square of a moving particle's velocity $v$. In this case, the distances covered by the moving body would turn out to be proportional to the velocities. Consequently, different distances would be covered in equal times to mean that the moving body would occupy different places at the same time[23]. Once again one should be well aware of this „non-locality“ as a curiosity that is *not* a property of nature, but rather results from the application of a *taken for granted* a priori foundation of the theory on a „squared“ energy term[24]. Obviously this term must be abandoned in favour of the QM „linear“ energy if one wants to obtain a *con-sistent* QM theory of motion or momentum as *effect*, and of energy as *cause* of this effect, and of the true energy-momentum *proportionality* to reliably separate effected momenta spatially and temporally from their generating causes, and from each other[25].

## <u>5. On a general quantum theory of motion implying SRT and gravitation</u>

Quantum mechanics and *Einstein's* special theory of relativity (SRT) show closely related already according to the above considerations. One only needs to realize the above demonstrated basic proportionality $E/p = c$, or $E = p \times c = (mv) \times c$, to represent the background of *Einstein's* famous equation $E = mc^2$. As it has already been shown by *Max Born*, an elementary „non-relativistic“ derivation of *Einstein's* equation inevitably yields the result $E = (mv)c$ [26], i.e. our eq. (2). And this result of course must lead to $E = mc^2$ if one only substitutes for $(mv)$ the product $(mc)$ that represents the measure of the momentum $p$ of light. *Einstein's* $E = mc^2$ then appears as only a special case of the general law of motion $E = p \times c$, which special case describes the motion of light particles *m* to move with the vacuum velocity of light *c*, i.e. with a momentum $p = mc$ (cf. paragraph 2.2.). The *correct* (i.e. physically meaningful) representation of the Einstein equation then reads $E/mc = c$, or $E \propto mc$, or $E \propto p$, contrary to the usual $E/m = c^2$ -interpretation which mostly, by the way, is not understood as a *proportion-ality*, but rather, and erroneously, is set up as an *equivalence* of mass and energy which evidently *is not* the mathematical contents of this equation[27)].

*The quantization condition*, for mechanics as well as for SRT, then is easily given if one only adopts *Newton's* authentic view that the factor *m* (i.e. „mass") always represents an integer multiple of one most elementary particle, so that the measure, or the *dimension* of *m* is (1,2,3...n). And this same Newtonian concept *of „mass" as the quantized measure of matter* also works as a simple and convincing tool to construct what has been haunting modern physicists ever since: *a quantum theory of gravitation*[28)]. For this reasoning, cf. also paragraph 6.1.

## 6. What's new in QM ?

6.1. The aim of QM to represent the quantities of physical entities such as *momentum* and *energy* as integer multiples of elementary quanta corresponds with *Isaac Newton's* authentic mathematical concept of „mass" *m*. Historians of science do know that *Newton's* definition of *m* as „quantitas materiae" means a macroscopic *discrete* multiple of equal microscopic constituents of matter, say „atoms" in the sense of the Ancients[29)]. Consequently, the quantization of physical entities which imply the quantity *m* results *immediately* if one only accepts *Newton's* quantized concept of *m,* starting with $p = mv$ $(m = 1,2,3...n)$. And this suggestion will obviously hold as well for a quantum theory of gravitation.

6.2. New QM concepts („new" with respect to classical mechanics) certainly are the „linear" energy terms, $E = hn$, *and* $E = pc$ the latter of which shows, and the former of which implies, the strict proportionality between energy *E* and momentum *p*, tied together basically not by *Planck's h*, but by the constant *c*. Since some entity *E* obviously cannot stand to another entity *p* in a *linear*, and in a *squared* relation *as well* (which follows from *Euclid's* axiom 1[30)]), it is clear then that the QM concept of „linear" energy means something different from the classical „squared" concept of energy. And QM „linear" energy indeed means the proportional non-phenomenal *generating cause* of true motion. Thus the come-back principle of *causal* *generation* is revealed as a characteristic of nature, clearing the way for a realistic philosophical conception of the principle of free will (cf. footnote 34).

*Erwin Schrödinger* alone was well aware of the incompatibility of energy concepts used in QM, even though he never explained the difference between QM and classical „energy" *mathe-matically* [31], as it has been done here for the first time, by means of geometric proportion theory[32]. It should, however, be stressed that the *E/p* proportion revealed here as the QM basic principle, has its precursor - in *Newton's* second law, correctly interpreted according to proportion theory, as I have shown it elsewhere[33].

6.3. What's new, then, in the QM theory of motion? Nothing at all, if we only take *Newton* at his word. As a matter of fact, QM implies a rebirth of some elements of the authentic *causal*, but *indeterministic*[34] Galileian-Newtonian quantum theory of motion which was buried for long under the so-called „classical mechanics", say a heap of Leibnizian dogmata such as „natura non facit saltus"[35] (in order to establish against Newtonian atomism a *continuum theory* of everything), and „causa aequat effectum" (in order to eliminate incommensurables together with proportion theory from mathematics and physics, in favour of only *functional* and *instantaneous* arithmetic relations or *equations* between *equivalent* entities, and in order to establish a rationalist and materialist theory of motion *on material observables only* [36]).

Up to today it has been widely believed that QM were an enigma not to be understood according to the usual way of understanding, that is by reducing its terms to known principles. As we can see now, this was a false doctrine. Nature again turns out to be a book man can truly read, provided he understands the language of timeless true geometry.

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Footnotes and references.

1) *Max Planck*, Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum, Verhandlungen der Dt. Physikal. Gesellsch. 2, p. 237-245 (14.12.1900).

2) *Albert Einstein*, Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt, Ann. Phys. 17, p. 132-148 (1905).

3) *Isaac Newton*, The Principia, *I.B. Cohen* and *Anne Whitman* eds., Berkeley etc., 1999, p. 382. Dt.: Mathematische Grundlagen der Naturphilosophie, *Ed Dellian* ed., Hamburg 1988.

4) *Werner Heisenberg*, Physikalische Prinzipien der Quantentheorie, Leipzig 1930, p.1; 44.

5) *Isaac Newton*,The Principia, p. 404 (definition 2).

6) *Werner Heisenberg*, Physikal. Prinzipien, p. 93. According to *Max Born* it was *John Henry Poynting* who already in 1884 derived from *Maxwell's* eqs. not only for light, but for *light-matter interactions* (such as the light-pressure phenomenon) the energy-momentum relation $E/p = c$; cf. *Max Born*, Die Relativitätstheorie Einsteins, Berlin etc., 1984 p. 244.

7) *Euclid* V, 8 on the trinomial proportion as the shortest one.

8) The symbol $\propto$ together with proportion theory on the whole has nearly completely vanished from science. It is hardly to be found in the literature on QM, and it is absolutely unknown in nowaday mathematics and logic. The reason, in my view, is that *proportions* indicate a *non-logical*, but *onto*-logical or *real* a- priori feature or *structure* of nature.

9) Cf. *Ed Dellian*, On Cause and Effect in Quantum Physics, Spec. Sci. Techn. 12 Nr. 1 (1989) p. 45.

10) This is the famous *de Broglie* relation; cf. *Werner Heisenberg*, p. 4.

11) *Max Planck* (1900), see ref. 1).

12) I want to stress the point that to my knowledge the „enigmatic" (*Max Jammer*) structure of QM has never before been analyzed by means of geometric proportion theory. Exceptions are my own publications (cf. ref. 9, and „Does Quantum Mechanics Imply the Concept of Impetus?", Physics Essays, 3 Nr. 4 (1990) p. 365), and *P. Kirschenmann*, Reciprocity in the Uncertainty Relations, Philos. Sci. 40, Nr. 1 (1973) p. 52-58; see also *Max Jammer*, The Philosophy of Quantum Mechanics, New York etc., 1974, p. 80.

*Kirschenmann's* mainly true objections against the general application of proportion theory
to the Heisenberg relations, however, do not affect my approach, rather their result corresponds exactly with the view which I have expressed in paragraph 3.1.2.

13) *Werner Heisenberg*, p. 2; 3; 9 ff.

14) In my view, one of the foremost achievements of *Galileo* and his followers was to understand that *true* (i.e. undisturbed) motions such as uniform rectilinear, and uniformly accelerated motion, or e.g. the orbit of the earth around the sun, can be treated mathematically, and do represent a true reality *even though we cannot observe them in nature.*

15) If one analyzes a quaternary proportion, A:C= D:B, one finds that the product of the outside terms A, B is equal to the product of the inside terms C, D. Consequently, a quaternary *equation of products* A, B, and C, D, can be rearranged into a *corresponding proportion* A:C = D:B according to this rule.

16) *Erwin Schrödinger*, Quantisierung als Eigenwertproblem, Vierte Mitteilung, Ann. Phys. 81, p. 109-139 (1926).

17) Cf. *Haken-Wolf* , Atom- und Quantenphysik, Berlin etc, 1983, p. 117 („Die Schrödinger-Gleichung"), pp. 58,114,117 eqs. (9.22), (9.23). - I have been pointing to the incompatibility of „linear" and squared" energy-momentum relations as early as 1987; see Proceed. of the Internat. Workshop on Matter Wave Interferometry in the Light of Schrödinger's Wave Mechanics, Vienna 1987, G. Badurek, H. Rauch, A. Zeilinger eds., Amsterdam 1988, p. 394,395.

18) *Haken-Wolf* p. 117 eq. (9.24).

19) *Haken-Wolf* p. 117: „Was muss man tun, um $h^2k^2/2m$ aus $\exp(ikx)$ und $hw$ aus $\exp(-iwt)$ zu erhalten, so dass die Beziehung $h^2k^2/2m = hw$ gilt? Differenzieren wir $\exp(ikx)$ zweimal nach $x$ und multiplizieren mit $-h^2/2m$, so erhalten wir tatsächlich als Faktor die linke Seite von (9.23). Entsprechend ergibt sich die rechte Seite, indem wir $\exp(-iwt)$ nach der

Zeit differenzieren und mit i*h* mutiplizieren. Damit haben wir schon die grundlegende Schrödinger-Gleichung des kräftefreien Teilchens....“. - Of course it is always possible to mathematically „repair“ a mistaken equivalence by means of multiplication with different factors on both sides of the equation. For instance, 3 = 5 can be „repaired“ by multiplying with 5 on the left side, and with 3 on the right side, in order to obtain a consistent 15 = 15.

20) It is clear now that $E/p = v/2$ is equivalent to $E = p \times v/2 = mv \times v/2 = mv^2/2$ - to prove again that *Schrödinger's* equation rests on the concept of classical kinetic energy only.

21) Cf. *Erwin Schrödinger*, Über das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen; Ann. Phys. 79 (1926) p. 734-755.

22) As a matter of fact, most of the really very sophisticated mathematical presentations of QM result from the venture to treat *differently* defined entities (such as $E = hn = pc$ on the one hand, and $E = mv^2/2$ on the other) as *identical* - a task never to solve rationally. An early attempt of this type ($E = hn = p^2/2m$) shows *Albert Einstein*, Quantentheorie des einatomigen idealen Gases, Sitz.ber. Preuß.Akad.Wiss. math.-physikal. Kl., 1925, p. 3-14; *A. Pais,* Subtle is the Lord..., Oxford etc., 1982, p. 437; cf. also *Erwin Schrödinger*, Quantisierung als Eigenwertproblem, Zweite Mitteilung, Ann. Phys.79, p. 489-527 (1926).

23) Cf. *Galileo Galilei*, Discorsi, Leyden 1638, 3rd day, *Salviati* (on the proportion of velocity and time versus velocity and space, in the law of free fall). If a body moves with a velocity *v* to cover some distance *l*, and *v* should grow in proportion with *l*, then this body, moving with double velocity, 2*v*, would cover double that distance, 2*l*, *in the same time* that was needed to cover the distance *l* with velocity *v*: $v = l/t$; $2v = 2l/t$ ; ($t = t$). However, to cover a distance, *l*, and another distance, 2*l*, in the same time, *t*, would mean *to occupy different places at the same time,* q.e.d. And this is the reason why a theory of motion which presupposes a proportionality between energy *E* and the square of velocity,$v^2$, i.e. between velocity and space, inevitably must imply non-locality, and non-local actions at a distance.

24) Was it not *Albert Einstein* who, in 1926, in a conversation with *Werner Heisenberg* in

Berlin, casually asserted that „it is the theory which decides what we can observe" (cf. *Max Jammer* p. 57)? Which assertion marks a decisive step away from the objective methodology and achievements of the Galileian-Newtonian philosophy of nature onto relativism and subjectivism, and to the nowaday *trial-and-error* game to first „invent theories" (*Stephen Hawking*), and then look around (not for falsification, as *Karl Popper* once claimed, but) for experimental corroboration.

25) If any effected momentum $p$ to represent a body at a certain time at a certain place is *proportional* to its cause $E$, and the proportionality constant is a quotient of elements $\Delta l$ of space and $\Delta t$ of time, $E:p = \Delta l:\Delta t = c$, then this constant $c$ will spatially and temporally separate different momenta from their different causes as well as from each other.

26) Cf. *Max Born*, Die Relativitätstheorie Einsteins, p. 245, where the energy term $E = (mv)c$ *for matter* (!) appears (as a consequence of *Poynting's* finding of 1884; cf. footnote 6)).

27) It was indeed the inventor of $E = mc^2$ himself who, from 1906 on, distributed the mistaken mass-energy „equivalence"-interpretation; see *A. Einstein*, Prinzip von der Erhaltung der Schwerpunktsbewegung und die Trägheit der Energie, Ann. Phys. 4. Folge, XX, 627-633. One should see, by the way, that this erroneous mathematical interpretation of a mathematical relation as an *equivalence* cannot be healed by any later *experimental* finding of any matter-energy transmutations.

28) Besides the „quantization" problem, classical gravitation theory suffers from a most problematic „action-at-a-distance" appearance. This problem, however, will easily be removed if one introduces a concept of „force" which is equivalent to the QM „linear" energy concept, as it includes the constant $c$. Indeed, this constant, as a quotient of elements of space and time, does *separate* effects from their causes *spatially* and *temporally* (cf. footnote 25)). Actually, it is an indispensable constituent of *Newton's* authentic theory of motion and gravitation, as I have already demonstrated it elsewhere (cf. reference 33).

29) Cf. *Erwin Schrödinger*, „2400 Jahre Quantenmechanik", Ann. Phys. 3, p. 43-48 (1948). It happened only after *Newton's* death in 1727 that, in the course of the 18th century, mechanics, by drawing on the deterministic Leibnizian calculus, tacitly took over anti-atomistic

concepts of the Leibnizian philosophy such as the continuum, thus resulting in a „classical mechanics“ which has much more to do with the neoscholastic philosophy of *G.W. Leibniz* than with the true theory of motion of *Leibniz's* philosophical antipode *Isaac Newton*.

30) *Euclid* I, Axiom I: If a = b and b = c, then a = c. Consequently, if a = b, and b ≠ c, then also a ≠ c. This is certainly the most elementary rule of rational science.

31) *Erwin Schrödinger*, „Might perhaps Energy be a merely Statistical Concept?“, Nuovo Cimento, IX Nr. 1, p. 162-170 (1958); cf. *Max Jammer* p. 29, who quotes from a letter of *Schrödinger* to *Max Planck*, May 31, 1926: „The concept 'energy' is something that we have derived from macroscopic experience...only. I do not believe that it can be taken over into micro-mechanics just like that... The energetic property of the individual partial oscillation is *its frequency*.“ - The latter point refers to the *E/n* - proportionality which is present in *Planck's* equation *E = hn*. I want to stress the point that in QM nobody (except *Erwin Schrödinger* and me) has ever understood *Planck's* equation as an introduction of an energy term which (as it is proportional to the momentum *p*) *can by no means be identical* with the classical „squared“ kinetic energy. *Schrödinger*, however, never drew his conclusions from this insight that could have prompted him to revise his theory of 1926.

32) There are some precursory publications, though. Cf. *Ed Dellian*, as in footnote 12.

33) Cf. *Ed Dellian*, „Die Newtonische Konstante“, Philos. Nat. 22 Nr. 3 (1985) p. 400; „Experimental Philosophy Reappraised“, Spec. Sci. Techn. 9, Nr. 2 (1986) p. 135; „Inertia, the Innate Force of Matter...“, in *P.B. Scheurer* and *G. Debrock* (eds.), Newton's Scientific and Philosophical Legacy, Dordrecht 1988, p. 227-237.

34) One should be well aware that the philosophy and interpretations of QM are heavily plagued by the erroneous *identification* of „causality“ and „determinism“. Causality, contrary to determinism, originally (and for *Newton*, e.g.) meant *the existence and unforeseen effectiveness of generating causes*, the „active principles“ or „forces of nature“ such as the free

will of living beings. Cf. *Newton*, The Principia p. 944. The difference was a main issue of the philosophic controversy between *Isaac Newton* and *G. W. Leibniz*. See *Samuel Clarke*, Der Briefwechsel mit *G.W. Leibniz* von 1715/1716, *Ed Dellian* (ed.), Hamburg 1990.

35) Cf. *John von Neumann*, Mathematische Grundlagen der Quantenmechanik, Berlin etc., 1996, p. 4.

36) Modern physics has picked up this trail in so far as relativity, as well as QM, is characterized through the introduction of an „observer" into the theories, in order to measure physical quantities not „as they really *are*", but as they *appear relative to the observer* who represents the (*anthropocentric*) „system of reference". See also footn. 14) and 24), and cf. *A. Einstein*, Zur Elektrodynamik bewegter Körper, Ann.Phys. 17 (1905), p.897 (*Einstein's* definition of simultaneity); *Werner Heisenberg* p. 43, 44; *John von Neumann* p. 222-225. I share the view of *Erwin Schrödinger* (1958, p.167-70), in so far as he critizised this way, warning of „the peril of a progressive narrowing of our field of vision,a mental glaucoma as it were."