# Newton's Mechanics is Quantum Mechanics.

Abstract.

An analysis of Newton's second law of motion in the *Principia* (1687/1713) and of Galileo's view of generation of motion in the case of naturally accelerated motion in the *Discorsi* 1638) shows that "acceleration" results from an addition of equal increments of uniform straight-line motion, the integer number of which is generated in proportion to the integer number of discrete quanta of "impressed force" and of equal particles of time elapsed. The (Galilean-) Newtonian geometric theory of motion is a quantum theory of motion.

## I Once again: *Is Newton's second law really Newton's?* Including A Brief Comment on Bruce Pourciau's 2011 Paper in Am. J. Phys. 79 (10), October 2011, p. 1015.

1. About three years ago, a paper entitled "Is Newton's second law really Newton's?" by Bruce Pourciau was published in Am. J. Phys. 79 (10), October 2011. The author correctly states that Newton's second law as recorded in his *Principia* of 1687 does not fit with its modern textbook representation, which is the equation **f** = *m***a.** Consequently he raises the question "what exactly does the *Principia's* second law assert?" This question to answer (among others) is the author's aim, and he begins with a quote from the English translation of Newton's *Principia*, translated and edited by I. Bernard Cohen and Anne Whitman (Berkeley 1999):

> LAW II. *A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.*

What does that mean? The author rightly states that one should be able to answer this question with the help of Newton's definitions. Which definitions? Since the law speaks of a proportionality between two terms, a "change in motion" and a "motive force impressed", the author recommends "to look up Newton's definitions for 'change in motion' and 'motive force' in the *Principia*", but immediately continues asserting that "the definition for 'motive force' is confusing and the definition for 'change in motion' is missing."

Based on these assertions (or hypotheses) he concludes that it was "the unclear and incomplete account of the second law in the *Principia* that has left the intended meaning of the *Principia's* second law an unsettled question for over three centuries". And then the author proceeds to develop his own reading of the second law, mainly based, however, on ignorance as to (i) Newton's def. 4, and on ignorance (ii) as to Newton's Corol. 1 to the laws of motion, both (i and ii) ignored throughout Pourciau's paper. Moreover, the author recasts Newton's first law, giving it a meaning that should better fit with his own view, and then introduces an example (fig. 1 and 2), where he measures the magnitude of an impressed force applied on a moving body at P in the direction G not through the straight line PG (fig. 2), as Newton would do, but through the "deflection" LQ. This deflection he calls "the observed effect" generated by that impressed force, notwithstanding that, according to Newton, the observable effect is *a motion* of the body in the direction of the straight line PQ. With Newton's parallelogram of forces in the *Principia*, Corol. I, the author's proposal would mean to take as an effect the side BD of the parallelogram instead of the diagonal AD, and generally to replace Newton's *theory of motion* with the author's theory of "deflection". Ultimately, (back to Pourciau's fig. 2) the author, mistaking the deflection PG = LQ for the "effect", arrives at an absurd *identification* of cause "force" PG (= LQ) and effect LQ. All in all, the author introduces principles not of

Newton's theory of motion but of his personal theory of deflection. Since the author's theory is not the subject of my paper, I shall now move on elying on Newton's words only.

2. Does the famous *Principia* really give an "unclear and incomplete account" of the most basic second law of motion? Admittedly, Newton's book is not an easy reading, the more since the scrupulous reader, due to the notoriously often very limited reliability of translations, must tackle Newton's Latin. This should be done, however, in any case before criticizing Newton's account. In our case Newton's authentic Latin version of Law II shows that, contrary to Bruce Pourciau's claim, there is nothing "missing", "unclear" or "incomplete".

1) It is true that Newton (in his def. 2) defines "the quantity of motion", but not the "change" in motion (lat. *mutatio motus*). But this doesn't mean that a definition thereof is simply "missing". Actually Newton's own explanation to Law II gives a consistent interpretation, according to which the "change in motion" is the change $\Delta(m\mathbf{v}) = m\Delta\mathbf{v}$ in linear momentum, where $\Delta\mathbf{v}$ stands for a generated change in velocity or in the direction of motion.

2) The second term to confuse the author is the "motive force", as he calls it. But, had he only studied Newton's Latin, he would have learned that Law II speaks not of "motive force", but of an "impressed" force to cause a change in motion: the "vis motrix impressa". Note that "vis impressa" (Galileo's "impetus") was *a technical term* in Newton's days, and Newton defines it in def. 4, which definition the author generally ignores: The "impressed force" is clearly an action against a body that generates a certain finite change in the body's uniform straight-line motion, $\Delta(m\mathbf{v})$ again, in full accordance with Law II.

The different term "motive force" which the author stresses means something quite different. It appears only in Newton's def. 8, and only as an abbreviation for the here-defined "motive quantity" of the "centripetal force", which force is the subject of Newton's def. 5 – 8. The author correctly refers to this definition of "motive force" in section B of his paper. But contrary to the author's claim the "centripetal force" is *not* (as some others also erroneously believe) a certain kind of "impressed force", rather it works as a continuously active *"source" of* "impressed forces" according to def. 4 (throughout ignored by the author), where Newton's explains: "Est autem vis impressa diversarum originum, ut … ex vi centripeta". Which reads in English: "Moreover, there are various sources of impressed force, like … centripetal force" (Cohen-Whitman transl.). Therefore, *impressed force* (def. 4) *is not itself centripetal force* (def. 5-8), rather *it springs from* (the field of) centripetal force, just akin to spring water, which is not itself the spring.

After all, the meaning of Newton's authentic Law II is absolutely clear insofar as it states a proportionality between some force, which I here symbolize by the letter **K**, and a discrete change in motion, generated by that force as its effect, which effect is identical with the term $\Delta(m\mathbf{v})$ identified above. Thus we obtain the formula

$$\mathbf{K} \propto \Delta(m\mathbf{v}) \qquad (1)$$

as a true representation of Newton's Law II, in words: "The change in motion, $\Delta(m\mathbf{v})$, *is* proportional to the motive impressed force, **K**" (I omit the second clause of Law II here). There is no room and no need for precautionary arguments in support of the author's ungrounded claim.

3. Two questions remain, which the author doesn't raise. They concern the "proportionality" between the force **K** and its effect on motion, $\Delta(m\mathbf{v})$. What does that mean?

1) On geometric proportions.

We can put this problem more precisely by transforming formula (1) into an equation according to

$$\mathbf{K} = \Delta(m\mathbf{v}) \times \mathrm{C} \qquad (2)$$

where C represents the "constant of proportionality".

What would Bruce Pourciau say about this constant, in order to reveal what Newton's Law II really asserts? Nothing in detail. As a logician, he would tend to ignore it. Note that formal logic knows neither the symbol " $\propto$ " for "being proportional" nor the geometrical rationale of this special relation between natural entities. Therefore, it might be a consequence of the restricted power of logic that this logician in his footnote 19 writes the following:

"Because the mathematics of the *Principia*, for the most part, is a geometrical version of limits and calculus, Newton preferred to work with proportions rather than equations. But we lose nothing and we gain a more modern presentation treating these proportions as equalities".

What is said here about Newton's preference for geometry is true. But the author doesn't realize that this is mainly *geometric proportion theory*, present throughout the *Principia*, and he has no idea of the importance of geometric proportionality constants. This comes to light with his allegation that one could simply *drop* the constant of (geometric) proportionality C and thus "gain a more modern presentation", "losing nothing". This is as badly mistaken as if one, for example, would try to give the law of gravitation a more modern appearance by dropping the gravitational constant, *g*; or, as if one would reduce Planck's law $E \propto f$ (which is $E/f = h$ = constant) to only *an equality* E = f, or Poynting's $\mathbf{E}/\mathbf{p} = c$ = constant to an equality $\mathbf{E} = \mathbf{p}$, or to drop from the thermodynamic equation of states Boltzmann's constant *k*, allegedly "losing nothing" by dropping all these proportionality constants, *g, h, c, k*.

No further comment. But the case deserves a closer inspection, the result of which I will present in the following as short as possible.

a) Newton prefers geometry over arithmetic *because geometric proportion theory* in contrast to arithmetic and logic *allows for mathematical interrelations of natural quantities of a "different kind"* (cf. the Scholium after Lemma X, which Newton added to the second edition of the *Principia*). Quantities of a "different kind", or *heterogeneous* quantities, differ from each other in measures, or "dimensions". For example, if a quantity *a* of dimension [A] is related proportionally to a different quantity *b* of dimension [B], so that $a/b$ = C = constant, the constant of proportionality C will have the dimensions [A/B]. Now, should one drop this constant, one would obtain $a = b$. This is to say that one would have made the *heterogeneous* quantities $a \neq b$ of different dimensions, [A] and [B], appear as *homogeneous ones*, $a = b$, of same dimensions [A] = [B] = [A], somehow a case of an adding of apples and pears.

Therefore, to drop the constant of proportionality in eq. (2) would mean to arbitrarily and erroneously make the "force" and the "change in motion" *homogeneous* quantities of *same dimensions*. As a consequence, he who would simply put $\mathbf{K} = \Delta(m\mathbf{v})$ instead of $\mathbf{K} = \Delta(m\mathbf{v}) \times \mathrm{C}$ would change and mistake the mathematical contents and meaning of Newton's formula.

b) In his Preface to the Reader of the *Principia* Newton describes "the basic problem of philosophy" as the task "to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces". Since "forces" for Newton are "causes" (cf. his "De gravitatione …", def. 5) that generate motions, or changes in motions, as their "effects", Newton's Law II as presented above is the basic "law of causality", as a law of *generation of change in a body's state of rest or motion by a proportional cause "force"*. Just the geometric *proportionality* makes it a reasonable and effective instrument to perform Newton's research program. This instrument would be reduced to a tautology and thus destroyed, as soon as one would, by cancelling the constant of proportionality, present it as an *equation* "cause equals effect". Nobody could ever discover the cause behind a natural effect in motion were the measure of this cause nothing other than the measure of the observable effect itself.

b) There are many more arguments to show that the idea of making Newton's geometric proportions more "modern" by dropping constants of proportionality is mistaken.

Nevertheless it is true that the basic concept of "classical mechanics", the formula $\mathbf{f} = m\mathbf{a}$, shows exactly such an equivalence, or equality, of cause (force $\mathbf{f}$) and effect (rate of change of motion, $m\mathbf{a}$). But, if one carefully considers the origin of this formula, one will find that it stems not from Newton but from his philosophical antipode Leibniz. Leibniz conceived it as "dead force" in his "Specimen Dynamicum" of 1695, and indeed he conceived it as an *equality* of cause and effect, based on his self-invented principle "causa aequat effectum", a principle which, as has been said above, is nonsensical and absurd from Newton's (from the realist's) point of view. Therefore, the $\mathbf{f} = m\mathbf{a}$ formula is Leibniz's, not Newton's, even though a *similar* (but *not identical!*) concept can be found in Newton's def. 7 and 8 of the "centripetal force", to read $\mathbf{f} = [\Delta(m\mathbf{v})/\Delta t] \times C$, or, generalized, $\mathbf{f} = m\mathbf{a} \times C$. It is the proportionality constant C then which distinguishes Newton's "centripetal force" (as defined in *Principia*, def. 8) from Leibniz's formula $\mathbf{f} = m\mathbf{a}$.

c) It can be shown that Leibniz coined his "causa-aequat-effectum" principle after he had found in John Wallis's 1670 "Mechanica" the geometric proportionality between cause and effect as heterogeneous entities, resulting in a proportionality constant. This heterogeneity and this constant meant a serious obstacle in Leibniz's way to an arithmetic-algebraic calculus based on logic only, or on the basic mathematical principle A = A ("the whole contents of mathematics" according to the Logician Leibniz; see the Leibniz-Clarke Correspondence, Leibniz's second letter to Caroline). So he decided to make cause and effect *homogeneous at will* by *simply dropping* the constant of proportionality - in the same inadmissible manner as Bruce Pourciau recommends it to the modern mathematician (cf. for the historical background e. g. H. Breger, in: Leibniz' Dynamica, ed. Albert Heinekamp, Stuttgart 1984, p. 116, 118).

2) On geometric dimensions and their meaning.

A second most important question concerns *the dimensions* of the constant C.

a) As I have shown elsewhere (for example in Physics Essays vol. 16 (2003) no. 2), these dimensions are "element of space over element of time", [Δs/Δt]. Consequently, the full information contained in Newton's Law II is

$$\Delta\mathbf{K} = [\Delta(m\mathbf{v})] \times \Delta s/\Delta t. \qquad (3)$$

What does this mean? *It means that no change of state of motion or rest of a material body m can ever happen in another way than in space,* Δs, *and in time,* Δt. Therefore, "instantaneous" change (a change that doesn't consume time), which is an intrinsic – unrealistic - property of the reductionist and time-reversible formula $\mathbf{f} = m\mathbf{a}$ of classical mechanics, is not possible in Newton's true theory of motion.

*Moreover, Newton's law as represented by* eq. (3) *means that every change of state of motion or rest of a material body happens by discrete steps* $\Delta(m\mathbf{v})$ in *discrete times* $\Delta$t. So eq. (3) demonstrates that Newton's true mechanics is basically *quantum mechanics.*

b) The quantum-mechanical aspect of true Newtonianism can be proved mathematically. Just transform eq. (3) into $\Delta\mathbf{K} : \Delta(m\mathbf{v}) = \Delta s : \Delta t$. Replace $\Delta(m\mathbf{v})$ with $\Delta\mathbf{p}$ (momentum) and $\Delta\mathbf{K}$ with $\Delta\mathbf{E}$. This yields the quaternary proportion $\Delta\mathbf{E} : \Delta\mathbf{p} = \Delta s : \Delta t$, which can be transformed once more to appear as an equation of products

$$\Delta\mathbf{E} \times \Delta t = \Delta\mathbf{p} \times \Delta s \qquad (4)$$

(according to the rule "product of outside terms equals product of inside terms"). Note that this formula (4) as an equivalent representation of Newton's true Law II exhibits a striking equivalence with the Heisenberg relations of quantum mechanics written $\Delta\mathbf{E}\Delta t \geq \Delta\mathbf{p}\Delta s$ ($\geq$ h; note that h is implicitly present with the products). The admissibility of replacing $\Delta(m\mathbf{v})$ with $\Delta\mathbf{p}$ and $\Delta\mathbf{K}$ with $\Delta\mathbf{E}$ **(**showing among others Heisenberg's **E** as a vector quantity, equal to the "Poynting vector")**,** is corroborated by a general rule of modern physics. I quote it from Max Jammer's "The Philosophy of Quantum Mechanics" (New York, 1974, p. 54) as follows:

"The view that a formal identity between mathematical relations betrays the identity of the physical entities involved – a kind of assumption often used in the present-day theory of elementary particles – harmonizes with the spirit of modern physics according to which a physical entity does not do what it does because it is what it is, but is what it is because it does what it does. Since what it 'does' is expressed by the mathematical equations it satisfies, physical entities which satisfy identical formalisms have to be regarded as identical themselves, a result in which the mathematization of physics, started by the Greeks (Plato), has reached its logical conclusion."

c) Speaking now about quantum mechanics, it is quite clear that one arrives at a "Newtonian quantum theory of gravitation" as soon as one respects Newton's discrete concept of impressed force. By realizing that according to Newton's def. 4 gravity works as a *primary* cause to generate not instantaneously and continuously accelerated motion, *but series of discrete impressed forces* as *secondary* causes, which secondary causes according to eq. (3) generate in space $\Delta$s and time $\Delta$t *discrete increments of velocities, or momenta,* in the gravitating body, one immediately gains a *discrete* picture of the action of gravity. This picture is strongly supported by a paragraph which Newton inserted into the second edition of his *Principia* (1713). It reads (quote from the Scholium after Corollary 6 to the laws of motion):

"When a body falls, uniform gravity, by acting equally in individual equal particles of time, impresses equal forces upon that body and generates equal velocities; and in the total time it impresses a total force and generates a total velocity proportional to the time."

This description corresponds to *Principia,* Prop. 1 Section 2 ("To find centripetal forces"), where Newton shows how, by a successive series of discrete *impressed forces* to arise from the *centripetal force* as a source, a body's motion in a circle around a center can be performed. The only difference concerns *the direction* of the impressed forces and the body's motion. In the "circular" case the body is moving in the direction of a tangent to the circle when an impressed force acts on it in a different direction, that is, directed to the center of the circle. In the case of the falling body the impressed force and the body's motion point in the same direction "downwards".

As we see now, to respect Newton's concepts of centripetal force, or gravity, as a continuously active *primary* cause and a *source of* discrete impressed forces as *secondary* (immediate) causes to generate proportional changes in the motion of gravitating bodies brings forth the solution of a haunting enigma of theoretical physics of our time, eventually showing that the problem was home-made by ignorance of Newton's words.

d) A further achievement of recovering Newton's *true* Law II is that the annoying and absurd "time-reversibility" of Law II in its currently corrupted form, **f** = *m***a** vanishes with eq. (3). The said absurdity has often been criticized (cf. Oliver Penrose in Nature 438, 919; 2005), but to no other effect so far than that today many realists prefer the theory of thermodynamics because of its intrinsic time-irreversibility. But this characteristic results from the Boltzmann constant k as a proportionality factor only. The same effect would be achieved taking into account the proportionality constant C in Newton's authentic second law.

Therefore, aside from the occurring possibility of a unification of the true Newtonian theory of motion with modern quantum mechanics, the just mentioned achievements as to a Newtonian quantum theory of gravitation and to a realistic and irreversible law of causation of change in nature would strongly recommend the outlined improvement, even if doubts remained that eq. (3) should really represent Newton's intention. But during my lifelong study of nearly all of Newton's writings on natural philosophy (secondary literature included) I have found that any such doubts will be dispelled with a meticulous investigation of Newton's own words.

## II True Newtonian Mechanics is Quantum Mechanics (A letter to Bruce Pourciau).

Dear Bruce,

With email of 21 April 2014 I asked you whether or not Newton's second law, as it states a proportionality between "vis motrix impressa" (the motive impressed force) and "mutatio motus" (the change in motion), should require a proportionality constant. You immediately replied on 22 April, kindly admitting that "if one had the correct meanings of 'motive force' and 'change in motion', then one could write Law II as 'change in motion = k × motive force'". The letter k represents the proportionality constant.

I would now like to discuss Newton's Law II starting from that point of agreement. It was a pleasure to see us absolutely agreeing (as it seemed to me) on the requirement of a proportionality constant, and, generally, on the fact that Newton's authentic Law II does not correspond to the "force-equals-mass-acceleration" formula which the textbooks and even some eminent Newton scholars pass off to the unsuspecting public as "Newton's second law".

We agreed that the "correct meanings" required for a true understanding of Newton's law must be found in Newton's theory, not in secondary literature, where sometimes things are arbitrarily added to Newton's principles, such as an additional time derivative to Law II, and where generally the proportionality constant k is dropped (e. g. Max Jammer, Concepts of Force, 1957, p. 124). When some authors want to justify this dropping the constant, asserting that it would be just "mass", they ignore that mass is part of "motion" as defined by Newton, and therefore it is also part of the "change in motion" that is proportional to the force, so that mass is not available as a proportionality constant. When others are asserting that the constant would be a dimensionless number which could be put equal to "1" by a proper choice of units and consequently dropped (e. g. Jürgen Mitelstraß, Neuzeit und Aufklärung, 1970, S. 288), they tacitly presuppose that the proportional expressions would bear same dimensions, and thus they commit the mistake of a *petitio principii*.

The following is my position which I would like to discuss:

1. The discrete variable “vis impressa” of the second law is defined by Newton in def. 4 as an action on a body from outside to change the body’s state of motion or rest. This discrete “vis impressa” is *not* the continuous “vis motrix” which Newton defines in def. 8 as a quantity of the “vis centripeta”. Rather, the latter is a continually existing “source” (lat. origo) of the former (see Newton, *Principia*, the explanation to def. 4).

Newton’s concept of a specific discrete “vis impressa” is ignored and therefore absent in the works of many eminent Newton scholars. Some explicitly identify and fuse it together with the continuous “vis motrix” of def. 8; see for example Bruce Brackenridge, The Key to Newton’s Dynamics, 1995, p. 146; Francois De Gandt, Force and Geometry in Newton’s Principia, 1995, p. 17; Howard Stein, Newton’s Metaphysics, in: The Cambridge Companion to Newton, 2002, p. 286/7. Some others do the same implicitly, by merging discrete “impulsive forces” (impressed forces) with “continuous forces” *via the limit*; see Niccolò Guicciardini, Reading the Principia, 1999, p. 48; cf. Bruce Pourciau, Is Newton’s second law really Newton’s?”, AJP 2011, p. 1015 (1019); Michael Nauenberg, Orbital motion and force in Newton’s Principia … , 2014 (sect. 5).

Now what is the discrete “impressed force”, defined in Newton’s def. 4, and present as a most important concept in Law I and Law II? In Newton’s *Scholium* after def. 8 one reads that “impressed force” – and *only* “impressed force”! Not the “motive force” of def. 8!– *is the true generating cause of motion, and of change in motion*: “True motion is neither generated nor changed except by forces impressed upon the body itself” (*Princi*pia, the Cohen-Whitman translation, 1999, p. 412). Impressed force therefore is measurable through the measure of generated discrete quantities of motion, or change of motion, to which it is “proportional” (Law II).

2. The discrete variable “change in motion” is defined on the basis of Newton’s def. 2 of the “quantity of motion”. As this quantity is given through the product “mass times velocity”, m**v**, the change in motion is simply given through the expression $\Delta$(m**v**). The “proportionality” of this quantity to a quantity of “impressed force” is a well-defined mathematical interrelation between finite quantities of a different kind (Euclid, Elements, book 5, def. 6; cf. Newton, *Principia*, explanation to Law II: “If some force generates any motion, twice the force will generate twice the motion, and three times the force will generate three times the motion, whether the force is impressed all at once or successively by degrees”). Even though “change in motion” $\Delta$(m**v**) and “impressed force” are differently defined expressions to represent “quantities of different kinds” in the sense of Newton’s *Scholium* after Lemma X, there exists between them a geometric “proportionality“ as a rational mathematical interrelation. This proportionality will read *“impressed force” over “change in motion“ = constant*. Since the proportional expressions bear different dimensions, the constant must also bear dimensions of its own. Which ones?

3. The dimensions of the proportionality constant come to light when one for example considers Newton’s explication of the “first and last ratio” of just nascent or evanescent quantities of motion: Since at the very first beginning of the motion the generating forces **K** are proportional to the spaces described (while the relation of space over time squared = velocity **v** over time t = “acceleration” **a** is constant: Newton, *Principia*, Lemma X, Corol. 3), we obtain **K :** s

= **v :** t. This equation of ratios yields the generalized formula **K : v** = s **:** t = constant: The generating impressed force **K** and the generated velocity **v** are proportional to each other via a combining proportionality constant with dimensions "element of space s over element of time t". The same is true if one considers a generated *motion* m**v** and its interrelation with the generating force **K**: **K** = (m**v**) × k [s/t]. Note that this formula speaks of *discrete* quantities of motion, and consequently also of *discrete* proportional quantities of force **K**.

4. As a consequence of the geometric structure of Law II here comes to light the discrete *quantum-mechanical substructure of the Galilean-Newtonian theory of motion.* This substructure is evident when one takes Newton at his words concerning the discrete structure of space and time, in the *Scholium* after definition 8 in the *Principia.* It is also evident when we read a quote from Newton's *Principia* that refers to Galileo's law of free fall. Says Newton in the *Scholium* after Corol. 6 to the laws of motion: "When a body falls, uniform gravity, by acting equally in individual equal particles of time, impresses equal forces upon that body and generates equal velocities; and in the total time it impresses a total force and generates a total velocity proportional to the time". This process of a step-by-step generation of velocity and motion in the course of time, taking place by successive addition of generated increments of velocity (motion), is already present in Galileo's *Discorsi* of 1638, Third Day. Galileo introduces a figure which may also illustrate Newton's just quoted words.

The following figure 1 is Galileo's. It must be read from the beginning at A downwards. There are three successive "particles of time", AC, CJ, JO. The corresponding successively generated velocities of the falling body are CB, JF, OP. In every particle of time an equal discrete increment of velocity is added to the already generated velocity: In the time CJ the increment GF (= GJ) is added to the velocity CB (= GJ) already generated from A to C. In the time JO the increment QP (= GF = CB) is added to the velocity JF (= OQ) already generated from A to J, and so on. The figure also shows the step-by-step increase of the spaces described during the time of fall. The increments of space are given through the rectangles ADEC, ECHJ, BEGH, etc. Evidently in the course of linear succession of equal particles of time the corresponding number of these increments of space increases as the odd numbers: 1, 3, 5, etc.

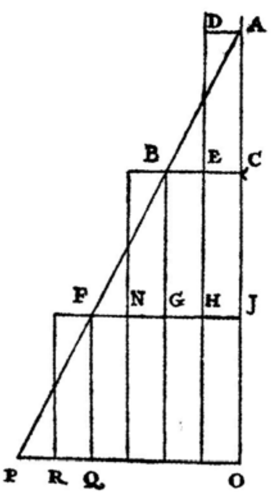


Figure 1: Free fall from A to O
(Galileo and Newton)

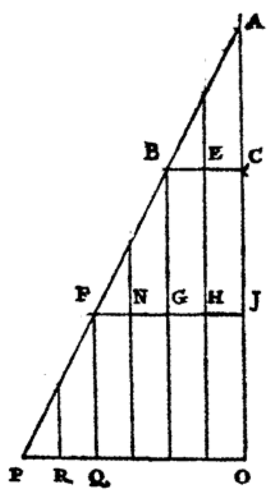


Figure 2: Free fall from A to O
(classical continuum mechanics;
Euler-Lagrange).

Figure 2 shows how *classical mechanics* (*continuum* mechanics) describes the same (but *only apparently* the same) phenomenon. The straight line AP represents a constant relation between a seemingly *continuous increase* of velocity and an also *continuous increase* of time: Velocity **v** over time t = "acceleration" **a** = constant, or motion m**v** over time t = "mass-acceleration" m**a =** constant. The latter is an expression of the continuous "vis motrix" (*Principia*, def. 8) that is called "motive force" in English secondary literature. Figure 2 makes evident the "parallelism" of this continuous force with a continuous increase of velocity, as it is the contents of the equation "**F** = d(mv)/dt = m**a",** the (only) concept of "force" of classical mechanics, where it, as a continuum theory of mechanics, is based on. The force is made continuous by *identification* with continuous acceleration via the equals sign.

The presupposed *continuous increase* of velocity in continuum mechanics (figure 2) is taking place in time ACJO according to the straight line AP that leads from null at A over BC at C and JF at J down to OP at O. This straight line is the locus of all velocities, infinite in number, as they should continuously emerge with parallels to BC. To every single instant of time, however minutely conceived, there "instantaneously" corresponds a full fledged quantity of motion. No *generation in time* of that quantity takes place. Remarkably, the areas of the triangles ABC, AFJ, APO, that is, the *spaces described,* which areas in figure 1 equal the area of the sum of rectangles described in the same time (rectangles that show the stepwise increase of space described), are in figure 2 the same as in figure 1. This is also true in all cases of similar triangles, even though the difference in the mode of increase of the space described is evident: stepwise and discretely in figure 1, continuously in figure 2. Therefore, classical continuum theory (figure 2), when calculating the spaces described (that is, the areas within such triangles) by ½ of the product of any triangle's short sides, yields exactly the results one obtains by adding the corresponding areas of rectangles to represent the spaces described in figure 1.

6. Assuming that the Galilean-Newtonian picture of a stepwise generation of velocity **v** (or motion m**v**) as shown in figure 1 corresponds to reality, this process must obey a condition formulated by Galileo: The moving body, before it acquires a particular generated velocity **v,** must acquire and stride across all the velocities smaller than **v** during the time of generation of this velocity. The following figures 3 and 4 will show this generation of velocity (motion) in time, as it forms the basis of the Galilean-Newtonian theory of motion.

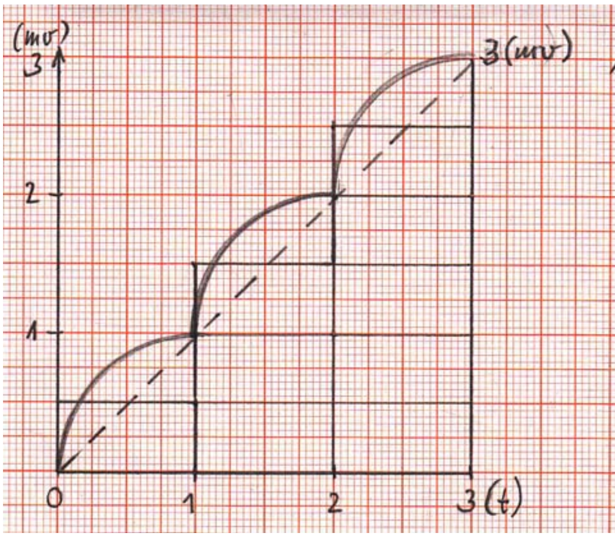


Figure 3

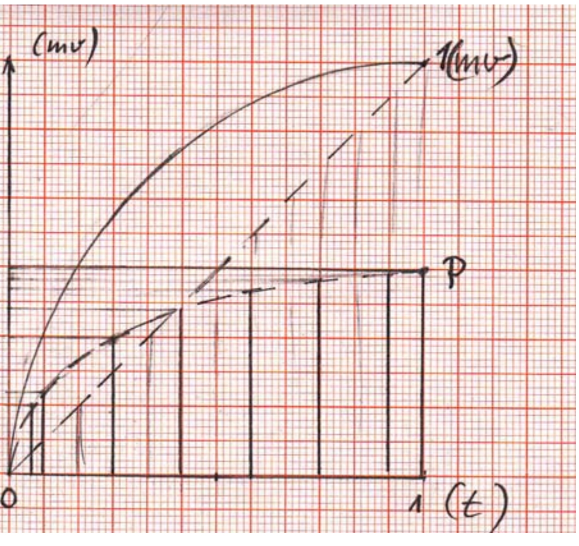


Figure 4

Figure 3 shows how a discrete quantity of motion 3(m**v)** is generated in three steps, that is, in three equal particles of discrete time t, beginning at zero. The space described in the first individual particle of time is given through the rectangle that is ½ of the area of the square over this time. Therefore, the whole space described during the generation of motion 3(m**v)** in time 3(t) is given through ½ of the area of the square over time 3(t).

Figure 4 shows the generation of motion 1(m**v)** in time 1(t), that is, the first step, at the beginning of the motion. It also shows the generation of space during this process, which space is given through an area continuously increasing in time along the curved line OP; and eventually it will be given through the area of the rectangle OP, which area is equal to ½ of the area of the square over time 1(t).

Going back to figure 3, one sees that the areas which represent the spaces described from time 1(t) to time 3(t) are in the ratio 1, 3, 5 etc., just as in figure 1 and figure 2, even though the ratio **v** over t, or the equal ratio of the spaces over the square of times (the "acceleration"), is evidently not constant in this figure 3, rather it decreases from a maximum at the very beginning of generation of every increment of velocity to a minimum when the generation is accomplished.

7. Figure 4, showing the generation of motion *in the very first particle of time*, also shows the situation "at the very beginning of the motion" in the sense of Newton's words in Lemma X of his "method of first and ultimate ratios". Two curved lines, one representing the discontinuous development of the acceleration **v/**t, the other one representing the also discontinuous development of corresponding areas (space described), when they approach the zero point will ultimately become equal, so that the space described will be in a square ratio to the time elapsed (Lemma X). But, since the relation **v**/t reaches a minimum when the generation of the increment of velocity is accomplished at 1(m**v**), a "constant average acceleration" should generate the same increment of velocity in the same time. This average acceleration is given through the straight line from zero to 1(m**v**) in figure 4, which is also the straight line from zero to 3(m**v**) in figure 3, and also the straight line from A to P in figures 1 and 2. Nevertheless, the realistic Galilean-Newtonian figure 1 shows the only true and real measuring points of equal average acceleration to lie at A, B, F and P in figure 1, exactly "in the middle" between the maximum and the minimum, that is, each time "at the very beginning of the motion only", namely, of the motion that is generated in time according to a *discontinuous development* of the velocity-time relation, the velocity, the time, and the spaces. The discontinuous development of spaces appears in Galileo's more schematic figure 1 *through the areas lying outside the line* AP (ignored in the "classical" continuous figure 2), while in figures 3 and 4 one has it before one's eyes with the arcs that connect the points of the discontinuously developing velocity-time relation from maximum to minimum. In this case, the corresponding spaces described are given through half the area of the square which is crossed by the respective arc (this half is a rectangle, cf. the rectangle OP in figure 4).

I have already tried to explain this process of a first generation of motion in a first particle of time with a figure more than 25 years ago, in my essay "Inertia, the innate force of matter, a legacy from Newton to modern physics", in P. B. Scheurer and G. Debrock eds., Newton's Scientific and philosophical legacy", International Archives of the History of Ideas, vol. 123, 1988, p. 232. Here is the figure:

Figure 5:

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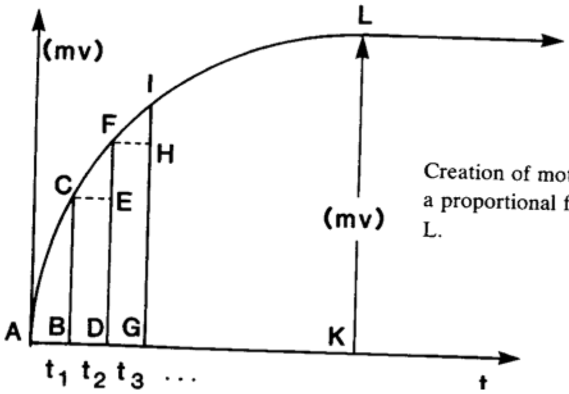
will be characterized by a continual diminution of the increase of motior (or of the velocity of the creation of the motion) which starts from a max-imum. The diagram below clarifies this description:




Creation of motion $mv = KL$ in time AK by a proportional force $mvc$ decreasing from A to L.

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8. Now one understands why Newton in Lemma X says that "the spaces which a body describes when urged by a finite force, whether that force is determinate and immutable or is continuously increased or continuously decreased, are in the squared ratio of the times *at the very beginning of the motion only"* (my italics). The most important word "only" is clearly required for a correct translation of Newton's Latin "ipso motus initio" into proper English. There is no "constant acceleration" *continuously* at work in the phenomenon of generation of "naturally accelerated motion" (Galileo): It is not a "uniform acceleration" but rather a *uniform addition of uniform discrete increments of uniform motion proportional to the elapsed equal particles of time* (as shown in figure 1 and figure 3) that characterizes this kind of motion. And, the same is also true in the case of *circular* motion, as Newton shows it in the *Principia*, Proposition I, Theorem I, where the uniform addition of uniform discrete increments of uniform motion is given as a uniform addition of uniform increments *of changes in the direction* of uniform motion in equal particles of time.

Conclusion:

1. According to Newton's Law II discrete quantities of "impressed force" generate discrete quantities of motion, so that "twice the force will generate twice the motion, and three times the force will generate three times the motion, whether the force is impressed all at once or successively by degrees" (Newton): To every degree of motion m**v** generated in a discrete time t there corresponds a proportional discrete quantity of "impressed force" **K**. Therefore, the impressed force is basically given through the formula $n\mathbf{K} \propto n(m\mathbf{v})$, which is equal to the expression $n\Delta\mathbf{K} = n\Delta(m\mathbf{v}) \times k$ (n = 1, 2, 3 … ; k is the proportionality constant). Geometric proportion theory, as taught by Euclid (Elements, book V, def. 1-6) inevitably requires a *stepwise* increasing of proportional quantities according to the succession of natural integers.

Down to the present day many scholars have been trying to interpret Newton's *discrete* theory of motion as to make it correspond with the principles of the Eulerian-Lagrangean classical *continuum* mechanics, characterized through a continuously accelerating constant force, as shown above (figure 2). In order to identify in Newton's *Principia* the mathematical expression of such a motion-generating force they simply and erroneously have replaced Newton's concept of discrete impressed force with his concept of the quantity of continuous centripetal force called "motive force". This motive force, however, is given in Newton's theory not through actual motion, but rather as the continually existing *endeavour of a body* m *toward a center only*, which here on earth is simply the body's "weight" (cf. *Principia*, def. 8, explanation). Its measure is proportional to $\Delta(m\mathbf{v})/\Delta t$. But the *quantity of generated real motion* of the body, be it in a circular orbit, be it in free fall downward, is always only given as the effect of a proportional series of discrete impressed forces that rise from the motive quantity of centripetal force as their source, in proportion to the number n of discrete particles of time $\Delta t$ elapsed (figures 2 and 3): $n\Delta t \times (m\mathbf{v})/\Delta t = n(m\mathbf{v})$ , with n = 1, 2, 3 …. .

2. In Newton's theory *the generation of* motion in time is a central subject. If a body is urged by an impressed force to change its state of rest or motion, in the course of this change a particular quantity of velocity is generated, but the body must pass through all the smaller velocities, or degrees of velocity, before it attains the particular velocity that is proportional to the generating force. Therefore, there takes place *a process of generation* of every discrete quantity of velocity or motion (unknown in classical mechanics), which process cannot take place *instantaneously* but only in time. This is shown in figures 1, 3, 4 and 5 (figures 4 and 5 giving an insight into the generation during *the very first particle* of time, and into the conditions "ipso motus initio", as Newton says, that is, at the very beginning of motion only). In this Newtonian picture, the ratio "velocity over time" evidently attains *at the very beginning of the motion only* a measure which *only at the very first moments of generation of motion* (at the measuring points ABFP in figure 1, and 1, 2, 3 in figure 3) is given through always the same (or constant) ratio "space over time squared" (Newton, *Principia*, Lemma X).

The true meaning of Newton's "method of first and last ratios" is exactly what the name of this method tells: It is a theory *of the first and last ratios <u>only</u>* of generating forces and generated motions (changes in motion). In a way the simple Latin word "ipso" in Newton's Lemma X (meaning "exactly", or "only" at the beginning of motion in this context) marks the abyss between the realistic Galilean-Newtonian quantum theory of true natural motion and the Cartesian-Leibnizian-Eulerian-Lagrangean continuum mechanics, which is a mere abstract, unrealistic construct of the human brain. It works in calculations, as has been shown; but it fails as a description of reality.

As far as "instantanity" is concerned (which is an intrinsic unrealistic property of continuum mechanics), Galileo and Newton knew very well that actually nothing takes place but in space and in time. Therefore, space and time, or "Newtonian spacetime", which is the discrete spacetime structure of reality, underlies, as a natural reference system, the proportionality of discrete generating impressed force and discrete generated motion. This structure comes to light with the proportionality constant of dimensions "discrete element of space over discrete element of time". It is this constant that governs Galileo's and Newton's most basic causal law of generating force and generated motion. It is this constant that, under the name "vacuum velocity of light", governs all of modern physics. It is this constant that binds together discrete quantities of heterogeneous entities such as generating "impressed force", or "energy", and generated motion, or momentum, in a rational mathematical form called "geometric proportion theory". Insofar as, remarkably, an eminent Newton scholar like Niccolò Guicciardini as "a modern reader considers the geometry of conic sections and proportion theory useless" (Guicciardini, 1999, p. 260), it must have escaped him that *geometric proportionality characterizes the most basic principles of modern science*, for example: Poynting's E over p = c = constant; therefore $E \propto p$; Planck's E over $\nu$ = h = constant; therefore $E \propto \nu$; Einstein's E over mc = c; therefore $E \propto mc$; and Heisenberg's $\Delta E \times \Delta t \geq \Delta p \times \Delta s$, resulting in $\Delta E/\Delta p = \Delta t/\Delta s$ = constant = c; therefore $\Delta E \propto \Delta p$ (vector notation throughout omitted).

3. The meaning of Newton's Lemma X has been obscured by many Newton scholars in the past, who misinterpreted Newton's method of first and ultimate ratios in a sense as if he had demonstrated that apparently discrete processes of change in motion would turn out to be *throughout continuous in reality* when analysed "at the limit". The result was a general mistranslation of the Galilean-Newtonian theory of discrete generation of motion into a Cartesian-Leibnizian continuum theory, implying the concept of a timeless, continuous and "instantaneous" emergence of (particular quantities of) motion, a concept that must be judged absurd from the Galilean-Newtonian *realistic point of view*. This judgement concerns the whole non-geometric "analytical mechanics" developed on unrealistic Cartesian-Leibnizian continuum principles by the Leibnizians of the $18^{th}$ century, by Euler and Lagrange, which theory today governs all physics textbooks in the world under the false name "Newtonian mechanics".

When Newton's name was misused and his theory corrupted as to describe, and to match with, the continuum theory of motion of his philosophical and mathematical antipodes Descartes and Leibniz, this meant to conceal the most serious scientific mistake and the most momentous paradigm shift of the past four centuries. But thanks to the rise of modern quantum theory the day will come when this *proton pseudos* of the scientific age will be corrected: when everybody will see, know, and understand what Galileo and Newton already knew, that only geometric proportion theory, and generally Euclidean geometry, provides the proper language he must learn who wants to *really understand* the language of Nature and the meaning of a true realistic theory of motion. The Galilean-Newtonian theory, understood in the language of geometric proportion theory, is *quantum mechanics*, and this to realize is a first step on the road to reality, part of "some fundamentally new insights that are certainly needed", as for example Roger Penrose admits (The Road to Reality, 2004, p. 1027). It is the key to a true understanding of the relation of modern physics to the reality of Nature.

30 June 2014. Ed Dellian, Berlin, Germany.

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