The Language of Nature is Not Algebra.

An Essay on the hidden power of geometric proportion theory as a tool of natural science and philosophy.

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ΑΓΕΩΜΈΤΡΗΤΟΣ ΜΗΔΕΊΣ ΕΙΣΊΤΩ

Off Limits to Non-Geometers!

(Written at the door of Plato's Academy in Athens, ca. 380 BC; also written at the Front Page of Copernicus's "De revolutionibus orbium coelestium", 1543 AD).

Abstract

In his book "Causality" (Cambridge University Press, 2000, 2009) Judea Pearl asserts that the language of Nature is algebra. Starting with a critique of Pearl's historical references I show that the language of Nature has always been geometry (even though algebra became the language *of natural science* after Newton). Geometric proportion theory comes to light as the original tool of scientific causal research. It is shown how this geometric approach was left behind when after Newton the mainstream science of Leibniz, Euler, Lagrange and Laplace turned to algebra. It is also shown that the endeavours of some modern scientists (Judea Pearl, Roger Penrose, Robert Rosen) to express causal relations by means of algebra have unwittingly resulted in the production of inadequate pseudo-algebraic, actually geometric proportion theories such as the symplectic geometry of Roger Penrose, and the relational theory of Robert Rosen. This result corroborates the view that the natural language of causal research must be geometry (geometric proportion theory).

Key words:

action, algebra, asymmetry, causality, energy, force, geometry, Heisenberg relations, heterogeneity, Newton's laws, Noether theorem, proportion theory, theoretical biology, tetraktys.

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I Introduction

Is there a language of Nature? Most readers would say, yes, the language of Nature is mathematics. Some would perhaps refer to Galileo Galilei (1564-1642), to his 1623 book “Il saggiatore” (the gold balance). But the reader would find that Galileo mainly speaks of *geometry*. Some others would think of Immanuel Kant (1724-1804), who in his 1786 book “Metaphysical principles of natural science” states that natural philosophy is *scientific only insofar as it is “mathematical”*. At that time the mathematics of science had turned from geometry to *algebra*. Today it is much more algebra, and geometry is somehow incorporated in it, and seemingly secondary. Is the language of Nature perhaps algebra?

Judea Pearl, 2011 winner of the Turing Award, is best known for championing the probabilistic approach to artificial intelligence and the development of Bayesian networks**.** The author of a demanding and sophisticated book on causality[1], Pearl believes algebra to be the language of nature (2009:405). To prove his assertion, Pearl refers to the “most profound revolution that science has ever known”, naming Galileo Galilei (1564-1642) as the “engineer” of that revolution. I will show in the following that Judea Pearl is wrong about this. It must be stressed, however, that this paper is only a “brief note” on Pear’s book insofar as Pearl raises the question of Nature’s true language to express causal relations. My general aim is to show some of the far-reaching consequences that follow from the correct answer to this question, and especially for the problem of causation.

Pearl when citing Galileo is referring to Galileo’s 1638 book known as the “*Discorsi*” [2] in which, according to Pearl, Galileo presents “two maxims: *one*, description first, explanation second; and, *two*, description is carried out in the language of mathematics, namely equations”. Pearl writes (2009:405):

“It is hard for us to appreciate today how strange that idea sounded in 1638, barely 50 years after the introduction of algebraic notation by Vieta. To proclaim algebra the *universal* language of science would sound today like proclaiming Esperanto the language of economics. Why would Nature agree to speak algebra? Of all languages? But you can’t argue with success. ”

Pearl is correct on the "introduction of algebraic notation by Vieta." Francois Viète, also called Vieta (1540-1603), a French lawyer and mathematician, is indeed renowned for having been the first to improve arithmetic calculation and the theory of equations by introducing and systematically using the *letter notation* which characterizes algebra. In 1591 he published his main work "Isagoge" (introduction to a new science). To be sure, arithmetic had been known as an *art of calculating* since ancient times, and Vieta did not extend his algebra beyond that range. Nobody in his day thought that method was a methodical means of natural philosophy. Since Antiquity, geometry and the theory of geometric proportions had been the only mathematical tools to be used in the realm of Nature. Renaissance natural philosophy at least since the time of Nicolaus Cusanus (1401-1464) was also related to geometry as the *art of measuring really existing things.* This situation remained unchanged until to the end of the 17th century [3] and beyond.

Pearl is also correct in that *modern science* in general speaks algebra. But he is mistaken in attributing this language to the natural philosopher Galileo. Galileo stood for geometry, not algebra, as the language of natural science [4]. So does also Newton, (1642-1727), a fact which is not so well known [5]. Nowhere in Newton's and Galileo's theoretical work on motion can one find an algebraic equation. Galileo was a died-in-the-wool geometer [6]. And so was Newton, who once called non-geometric approaches to nature "the method of the bunglers in mathematics".

Nevertheless it is an interesting idea to understand the evident turn to algebra after Newton as a cientific revolution, unnoticed so far. Actually, however, the "algebraic revolution" happened only in the course of the 18th century when Galileo and Newton were long dead. It was the result of a general acceptance of the mathematical work of Newton's personal enemy and philosophical opposite, Gottfried Wilhelm Leibniz (1646-1716), who had based his mathematics on an early attempt by René Descartes (1596-1650) to reduce geometry to algebra (1637) [7]. Leibnizian *algebraic mathematics*, based on the continuum theory of numbers, became the language of science during the first half of the 18th century, and Leibniz's *theory of motion* (Specimen Dynamicum, 1695) served as the algebraic foundation of what became "analytical", or "classical" mechanics thanks to the works of Leonhard Euler (1707-1783), Jean d' Alembert (1717-1783), Joseph Louis Lagrange (1736-1813), and Pierre Simon Laplace (1749-1827).

While one may say that the language *of science* thanks to Leibniz and Euler etc. became algebra the question remains whether it is justified to call algebra *the language of Nature herself*. In the following sections, we shall see that it is not. The reader will see that serious consequences follow from this correction.

II Causality - Some Errors, Some Mistakes, and a Basic Correction.

What is Judea Pearl's causality about? In the first sentence of his epilogue Pearl tells us that his topic is "our awareness of what causes what in the world and why it matters" (2009:401).

1. Pearl continues: "Causality, even though basic to human thought, is a concept shrouded in mystery, controversy, and caution, because scientists and philosophers have had difficulties defining when one event *truly causes* another." (2009:401). He puts the question whether the evident *connection* between such events as the rooster's crow and the sunrise (2009:406) is a *causal one*. Does the event of the rooster's crow *cause* the event of the sun to rise? Certainly not. But why not? What then is causation?

To think of causation as a mysterious influence of "events" on each other is an unwarranted hypothesis that is conditioned by a general belief in algebra to be the language of nature. And this can be demonstrated as follows.

1) "Events" are normally called phenomena, i.e. the observable natural changes of a given order of material objects, such as the falling of an apple from the tree. In the language of causation such an event was formerly understood as an *effect* of some other *cause*. It is well-known that Newton discovered the force that springs from the gravitational field to act on the apple from outside (the *vis motrix impressa* in Newton's originally Latin words) as the true *cause* of such an effect. Is this force an "event"? Certainly not. It is not a phenomenon, since it is not observable itself. For Newton it is not even a *material* something.[8]

2) As a consequence of Newton's natural philosophy, so long as we rely on it, the "chain" of causation doesn't consist of phenomenal "events" to act on each other, rather there are non-observable generating "causes" such as the force that acts on the apple, and observable material "effects" generated by those forces, such as the apple's falling from the tree. And the "chain" basically has only two links: the generating cause, and the generated effect, so that it

makes no sense, for example, to ask Newton for the cause of gravitation, because this would mean to ask for the cause of a cause. His often-cited reluctance as to this issue does not refer to the real existence of the force itself but only to the missing theoretical explanation of some *specific properties* of that force which he enumerates in the Scholium generale added to the 1713 edition of the Principia. [9]

The problem with "causation" then mainly consists in the problem to discover the non-observable "forces" that cause observable effects (that is, events, or phenomena) [10]. This problem was solved by the old geometers as they noticed that the effect must somehow quantitatively respond to its proper cause, so that the first step was to understand precisely the effect, and then to deduce from it the quantity of the proper generating force. For this quantitative inquiry a *mathematical* treatment was needed. But the quantity of effect could not be taken as being also the quantity of the cause, since the material nature of the substance of an observable effect differs from the non-material substance of the generating cause, as it is evident in the case of the falling apple. So long as "cause" and "effect" were understood as natural entities *of a different kind*, the quantities thereof, even though somehow corresponding to each other, could not be taken as *equivalents.* So they could not be put in an *algebraic equation.*

The solution is a *geometric* concept, the "proportionality" of quantities of a different kind. [11] A striking evidence is the proportionality of the force that causes gravitational motion, measurable in proportion to the weight of a material body, and the generated motion as its effect: double the force (double the weight) generates double the motion ("motion" measured "mass times velocity" according to Newton's second definition). The formal (mathematical) expression of this fact, or "law of nature", is to put the force A over its effect B which results in a factor C. This C is naturally always constant (the "proportionality constant"), because "proportionality" of A and B means that, as well as A/B = C, so also 2A/2B = C, 3A/3B = C, and so on [12]. So the quantity of the generating cause of an observable material effect could be measured according to "A = B times C" (C always being constant), provided one knows the quantity of C. This is not a major problem for the geometric task of measuring a cause. With respect to the dimensions of the variables A and B it is clear that the dimension of C must be [A/B]. Accordingly, "A = B times C" dimensionally yields "A [A] = B [B] × C [A/B]". There are many examples of natural laws to show that the constant quantity to relate variables with each other is a known natural constant. One could even say that *natural constants are always proportionality constants* [13].

3) The proportional relation of cause and effect as natural entities of a different kind is an insurmountable problem as a classical *algebraic equation*. Note that algebra, as it was originally based on binary Aristotelian logic, knows only mathematical relations between *homogeneous* entities (or variables) A, B, according to the recursion formula A = B = A to be expressed in an equation A = B. The term “equation” speaks for itself. So, if entity A differs in genus from entity B (entities A and B being called “heterogeneous” in this case), as in the case of apples and pears, or with causes and effects, and very clearly in biology, an algebraic or logical treatment of a relation between A and B is impossible. Galileo the geometer was already aware of the insufficiency of logic when applied to Nature. In our time, some successors of Galileo, theoretical biologists that are sensitive of the inadequacy of applying algebra to Nature, but alienated from geometry, have begun to look for a different algorithm [14].

In the 17th century Leibniz confronted a somehow inverse situation. Having become a devout Cartesian during his studies in Paris from 1672-1676, he believed in the Cartesian dualism of spirit and matter, both heterogeneous entities strictly separated from one another (in Leibniz’s view), so that no interference, no influence of spirit on matter, no spirit-matter interaction should take place. Accordingly, natural science should deal with the material world, with matter only. And here we are at the birthplace of modern materialism. [15] An analog of the Cartesian materialist reduction of nature was the reduction of geometry to algebra which Descartes had initiated in 1637. So Leibniz wanted to apply to material nature the new Cartesian *algebraic* algorithm. When having read in the work of John Wallis the theory of geometric proportion of cause and effect, he considered how to conceive an *algebraic* causal law. Leibniz decided to make “cause” and “effect” no longer *geometrically proportional but algebraically identical, in order to treat them as equivalents.* [16] He expressed this idea in the motto “causa aequat effectum” (the cause equals the effect). As soon as the law of cause A and effect B was put A = B, no “constant of proportionality” C could disturb the beautiful algebraic symmetry. Tacitly, however, this operation had as a consequence that cause and effect, now apparently being equivalents, were put on one and the same ontological level as material “events”. And, this is the origin of the recursive symmetric formula “force equals mass-acceleration”, which was basically developed by Leibniz [17]. In 1750, Leonhard Euler published his version of the formula, calling it his own discovery [18]. Euler did not mention Leibniz in his paper, let alone Newton, to whom this law thereafter was nevertheless incorrectly ascribed by others as being a representation of Newton’s second law of motion.

None of this can be found in Pearl's book. But it is necessary to mention it in order to explain why Pearl by his belief in algebra is mislead to consider causes and their effects as "events".

4) Of course Judea Pearl proceeds far beyond Leibniz's somewhat primitive idea of simply equating causes and effects. But Leibniz's defective approach to the issue of "cause" and "force", even though strongly criticized by Samuel Clarke [19] , prevailed and entailed that later scientists generally thought that in order to know the cause of an observable effect it would suffice to clearly understand and mathematically describe the effect.[20] It is in this context that in his 1917 essay "On the Notion of Cause", Bertrand Russell rightly states that the concept of causality doesn't appear in the (algebraic!) laws of classical physics "which are all symmetrical [that is, based on the reversible equivalence of cause and effect, namely on the symmetric formula "force equals mass-acceleration"], while causal relations are unidirectional, going from cause to effect". [21]

Therefore, Pearl's way of trying to solve the problem with the help of stochastic and probability theory can be seen as a consequence of the false belief that (a) causes and effects are equivalent "events" on the same ontological level, and that (b) a correlation among such events (if there is any) can only be found "approximately", yielding just "plausible" results to some degree of probability, or degree of truth.

2. Pearl's next mistake comes immediately after he erroneously has baptized Galileo the father of algebraic equations. In order to emphasize this view he writes: "The distance travelled by an object turned out to be proportional to the square of the time." Obviously Pearl implicitly refers to Galileo's theory of motion, that is, to Galileo's law of free fall, as an instance of accelerated motion, because *in uniform motion* the distances are proportional to the times only, not to their squares. But this is a minor thing. The major mistake is the fact that Pearl unsuspectingly cites a veritable *law of geometric proportionality* of heterogeneous quantities "distance" (or "space"), and "time", calling it an *algebraic equation.* On p. 407 he presents a slide 13 entitled "Galileo's strange language: Algebra". It shows a drawing of some curved lines, and at the bottom the formulae $y \sim t^2$, $x \sim t$, and $y \sim x^2$. On p. 404, where Pearl refers to this slide, he calls these very formulae "mathematical equations". This is evidently untrue. Rather, the formulae present *geometric proportions of heterogeneous quantities* (i.e. *variab-*

les), x, y, t, the t symbolizing “time”, the y standing for “space”, the x remaining unexplained (the symbol “ ~ “ is sometimes used as to designate “proportionality”).
The instance shows that Pearl is unaware of the difference between algebra, the machinery of which “does not discriminate among variables”, as he correctly says on p. 405, and geometry on the other hand, which does exactly this. In other words, geometry in contrast to algebra is able to mathematically relate variables of different kinds such as apples and pears, “spaces” and “times”, “causes” and “effects”.

For the sake of completeness, it must be added that on p. 428 of his Acknowledgments, Pearl tells the reader that slide 13 is “from ‘The Album of Science’, by I. Bernard Cohen (1980)”. Unfortunately, no such illustration can be found in Cohen’s book. As one of the most educated American historians of science, Cohen knew very well that not even Newton ever used algebraic equations in natural philosophy, and he would never have ascribed such things to Galileo. So slide 13 in Pearl’s book p. 407 must be an outright construct, and this the author confirmed in an email of 18 June 2012, admitting that he had produced the slide himself.

3. Here comes the next hurdle. In his “brief historical sketch” on p. 405/6 (with a short remark on Leibniz, mistyped “Liebniz”) Pearl proceeds from Galileo directly to David Hume (1711-1776). The context is that scientists, who “had taken very seriously Galileo’s maxim ‘description first, explanation second’”, were taught by David Hume “that the *why* is totally superfluous as it is subsumed by the *how*”. In other words Pearl says here what we already have met with: the idea developed in the wake of Leibniz that it should suffice for science to describe the “effects” instead of asking for “causes” (cf. the d’Alembert quote in footnote 20). The important thing here is that Hume, who wrote his works after Newton’s death, doesn’t mention Newton either, nor does he refer to Newton’s theory of causation by nonmaterial natural causes (forces) in proportion to their material effects. So one should perhaps ask here why this is so, instead of restricting oneself to assert that “Hume argued convincingly” (Pearl p. 406). This is what Hume actually and evidently did not do, as he ignored the most important and effective theory of causality ever taught – a theory that explained the “why” of the true motion of the earth as well as the “why” of the moon’s remaining in her orbit, and the “why” of an apple to fall from the tree.

Pearl doesn’t totally ignore Newton. On p. 408 he presents to the reader as “Newton’s law” a symmetric “law of physics”. It is the formula “force equals mass-acceleration” ($f = ma$). This law is indeed symmetric insofar as it does not distinguish cause and effect from each other, but rather puts both things equivalently, well fitting with Leibniz's dictum "causa aequat effectum". Note that this formula stands in open contradiction to the words of Newton's cause-effect relation in the second law. An explanation of this inconsistency is lacking in Pearl's book. Remember what Sir Bertrand Russell correctly said (whom Pearl cites here): A symmetric formula cannot show a cause-effect relation since causal relations are unidirectional (asymmetric, that is), going from cause to effect.

Actually it is widely believed that Newton, having allegedly conceived "symmetric" laws of motion[22] such as the $f = ma$ formula, contradicted himself, and that he consequently held a very basic contradictory view in the idea of instantaneous action at a distance. It is true that this theorem follows from the laws of *classical* mechanics. But it doesn't follow from Newton's true theory of motion, provided one takes his words seriously. Note that Newton wrote to Bentley that well-known letter of Feb. 1692/3 in which he, with utmost strength, rejects action at a distance, calling it "so great an absurdity that I believe no man who has a competent faculty of thinking can ever fall into it."

Shouldn't this evidence urge a scientist to ask whether the alleged "reversibility" of Newton's laws might perhaps be due to a corruption that happened when these laws were no longer read in their original geometric form, but translated into the language of algebra by others? [23]

As Pearl presents as Newton's law the symmetrical (and thus actually "acausal") formula $f = ma$, I remember what I. Bernard Cohen wrote in his critique of Subrahmanyan Chandrasekhar's book "Newton's Principia for the Common Reader" (published 1995): "Readers should be warned that Chandrasekhar disdainfully and cavalierly dismisses the whole corpus of historical Newtonian scholarship, relying exclusively on (and quoting extensively from) comments by scientists, many of whose statements on historical issues are long out of date and cannot stand the scrutiny of critical examination. He falls into traps … such as … the form in which Newton expresses the second law. Chandrasekhar incorrectly equates Newton's 'change in motion' (or change in quantity of motion, or in momentum) with mass × acceleration…" [24].

I call the $f = ma$ formula "the mother of all scientific mistakes". But, as everybody knows, this formula is the basis of "classical mechanics" (which actually is a non-Newtonian continuum mechanics of Cartesian-Leibnizian-Eulerian proveniance). Thus, the formula is to some extent even the basis of all ventures *to overcome* the shortcomings of classical mechanics, from Carnot's and Mayer's thermodynamics (based on a scalar "energy" term being the space integral of $f = ma$) to the electromagnetic theory of motion of Faraday and Maxwell, to Einstein's attempts to unify classical mechanics and Faraday-Maxwell mechanics, and on to Quantum Mechanics.

4. It is revealing that, according to Pearl (2009:408), "the rules of algebra permit us to write this law ($f = ma$) *in* a wild variety of syntactic forms, all meaning the same thing – that if we know any two of the three quantities, the third is determined." Let us go into some details of this statement.

1) Why does Pearl speak of "three quantities"? Didn't we so far deal with only two apparently equal or equivalent homogeneous quantities, namely *cause* and *effect*, or "*force* and *change of motion*", or, if you like, "*force* and *accelerated motion*", placed equivalently, on both sides of an equation? But Pearl divides "accelerated motion", the *ma* term, into variables *m* and *a*, obtaining thereby a formula consisting of three *heterogeneous* terms, *f, m,* and *a*.

This is certainly something "the rules of algebra permit us". But this algebraic operation has dramatically misleading effects on the rational contents of the formula. For example, when written $f/a = m$ the saying is that "force" and "acceleration only" are related to each other, no longer "force" and "accelerated *motion*". And, when written $f/m = a$ the saying is that "force" and "mass" are related to each other so that double the force would correspond to (or cause?) double the mass. Clearly this all has nothing to do with Newton's second law that relates "force" to the effect "change of motion". The natural phenomenon "motion" is defined in Newton's second definition according to the product of mass *m* and velocity *v*, therefore one cannot put asunder the terms *m* and *v* and speak of "Newton's law of acceleration" (as Pearl does on p. 228) without destroying the rationale of Newton's law of nature, and even that of the non-Newtonian algebraic equation $f = ma$ which is to equate *f* and *ma* but not *f* and *a.* All in all, what "the rules of algebra permit us" can differ dramatically from *what Nature permits.*

2) Note that these transformations of *f, m, and a* only work if one of the three heterogeneous terms is made constant: $f/a = m$ works if *m* is made constant (the case when one and the same body m is considered); $f/m = a$ works if *a* is made constant (the case of uniformly accelerated

free fall of different bodies at the same rate). Accordingly both these cases represent what has above been explained as “geometric proportions” A/B = C = constant. Which is to say that those, who like Pearl use these transformations in order to show “what the rules of algebra permit us”, *tacitly transform the binary algebraic formula, an equation of two* homogeneous e*ntities, into a geometric proportion of three* heterogeneous *entities.*

This finding explains why if we know any two of the three quantities, the third is determined (Pearl 2009:54 knows the fact but not how it is explained): *It is the power of geometry alone* to demonstrate rational relations among quantities of different kinds (heterogeneous quantities, that is); and this power always and only takes effect when at least three terms are at hand, i.e. when the well-known “rule of three” can be applied. As we have seen, this rule is *geometric.*

3) Presupposing the equation $f = ma$ in the form $f/a = m$, Pearl admits that “the ratio *f/a* helps us *determine* the mass“ but notes that one doesn’t say that this ratio “*causes* the mass.” This expression reveals that he has no real insight into the “causal machinery”. It has never been asserted that *a compound ratio* (of force *f* and acceleration *a*) would itself represent a “cause”; rather it has been said that such a compound would itself show cause “force” and its effect *to stand in a defined relationship to each other.* Double the cause *f* would have double the effect *a*”, *m* representing the “constant of proportionality”, but never the “effect” of the quotient *f/a* as a ”cause”.

4) Pearl adds here that “such distinctions are not supported by the equations of physics”. He is evidently referring to *algebraic laws,* i.e. to equations A = B of homogenous variables A, B. But this means that he unintentionally admits that algebraic equations, being restricted to show relationships of variables of a same kind (homogeneous), cannot describe mathematical relationships among variables of a different kind (heterogeneous) such as “cause” and “effect”. In other words, in order to establish such a relationship based on algebra, cause and effect must *arbitrarily be made homogeneous variables* (this brings us back to the insight why Leibniz chose the motto “causa aequat effectum”). Otherwise it must be seen (in consequence of Pearl’s example of transformations of the $f = ma$ formula) that a true causal relationship between variable quantities, A, B, of a different kind *can only be established by embracing a third quantity, C, as a constant*. And this is nothing else but to dismiss algebra, and to form *a geometric proportion* according to A/B = C = constant instead.

5) Geometric proportions are traditionally represented by symbols " $\propto$ " or " ~ ". The proportionality of A/B = C is often expressed as "A $\propto$ B", or "A ~ B" which means the same in both cases. In both these cases the constant of proportionality is not made explicit; nevertheless it is implicitly present. Therefore, to replace the proportionality symbols by " = " (resulting in A = B) always means to arbitrarily omit the constant of proportionality, thus radically changing the meaning of the formula.

In algebra the symbol " $\propto$ " is unknown, and the same goes for the symbol " ~ ", which, it is true, sometimes is used to symbolize a "similarity". One must always keep in mind that similarity has nothing to do with proportionality of entities of a different kind. Interesting and revealing is the fact that *in formal systems of logic* there exists no symbol for "being proportional". Proportionality simply does not exist in algebra and in logical systems, for reasons explained above. And, as a consequence, Judea Pearl and most logicians and algebraists do not know what geometric proportionality is and are not aware of it when such proportions occur in mathematic formulae.

There are other little known symbols for proportionality. The Borromean Rings: Three flexible rings are so connected to each other that they all will fall apart if one of them is removed. Evidently any two of the rings are held together by the third one which connects them by connecting itself to them. This reminds one of Plato's definition of proportionality, as one finds it in his "Timaios" dialogue. There he speaks of the intimate conjunction of two things which can only be effected by a third one, a middle "bond producing the said most intimate conjunction by conjoining with the two conjoined things". And, this power Plato ascribes explicitly to the (geometric) "analogy" (which Greek term Cicero translated into Latin "proportio").

Another example is the tetraktys. Four terms, A, B, C, D, are mathematically connected to each other, forming what in English is properly called a "quaternary proportion". The four variables A, B, C, D can be rearranged as to show an equation of products $A \times D = B \times C$. Here it seems that between A and C the terms D and B are inserted to tie A and C together. But, by rearranging the terms according to $A/C = B/D$ and taking A and C "proportional" to each other, one sees that *the quotient* B/D represents the "constant of proportionality" as the "third Borromean Ring" to conjoin all the three, A, C, and itself, in a unity, indissoluble on principle. Plato gives an example when he speaks of God to have posed "water" and "air" in the middle between “fire” and “earth” so that the quotient of “air” (say, space s) to “water” (say, time t) conjoins “fire” (say, force, or “energy”, E) and “earth” (say, matter in motion, or “momentum p”) in a “friendly unity”. Evidently the chosen modern synonyms E, p, s, t form the well-known formula E/p = s/t = c = constant (the “Poynting vector” of energy flux density).

6) Note that the tetraktys is the formula behind the “rule of three”, and, as it has been demonstrated above, it is the only mathematical tool that allows gaining knowledge of the unknown on the basis of the known. Once again it must be stressed that this is a *geometric* formula, and, that *binary algebra* (corresponding to the binary Aristotelian logic) is not able without exception to do what the tetraktys can do, namely to extend human knowledge to what is unknown for the time being. This power, well-known to the Ancients, was recovered during the Renaissance, as it can be seen on Raphael’s picture “The School of Athens” (ca. 1510), on which painting a young man in the left foreground holds a table with the tetraktys drawn on it. Note that the well-known “golden ratio” represents as an archetype such quaternary proportions that are the core of the tetraktys. Evidently, however, this knowledge got lost when in the “Age of Enlightenment” after Newton, natural science was established on algebra, and Greek geometry was dismissed [25].

The attitude against geometry may be connected to the view, wide-spread at that time, of geometry to be “the language of God.” Therefore the “enlightened” scientists, when on good grounds dismissing the conjunction of science and church authority, dismissed also the geometric conjunction of empirical phenomena to non-empirical, or “metaphysical” causes (cf. d’Alembert as quoted in footnote 20). And this was not so well-grounded, so it seems to me. It ended for the time being in dismissing together with the geometric language of nature the relation of science with the reality of nature, establishing on the language of algebra a sceptical anthropocentric world view that should allow only for “conjectures and refutations” (Popper), and produce not true factual knowledge about nature but only probabilities and plausibilities [26].

5. As to the basic correction concerning the understanding of causality which has been announced in the title of this paragraph I must refer to the quote from Newton’s “Opticks” of 1717 introduced above. Newton explains his method of causal scientific research to consist in an *analysis of motions* to find “the forces producing them”. And in the Principia, in the au-

thor's preface to the first edition, he states that "the basic problem of philosophy [natural science] seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces". The reason is that there is always a phenomenon first, i.e. an observable effect, and the cause thereof is to be derived from the existing phenomenon. Or, the cause (the force) being known, an already existing phenomenon can be explained. But this method never makes it possible *to predict the future appearance of some new phenomenon.* Newtonian causality is *not* an art of predicting the future. It is an art to explain an already existing phenomenon by knowledge of its generating cause. So even the discovery that comets follow closed orbits, which made it possible to compute the date of their return, actually yielded an argument of reason against the belief in supernatural powers of seers and prophets. On the other hand, only a deterministic philosophy such as Leibniz's, who beliveed in a "pre-established harmony" (predestination, that is, namely the belief that God in the beginning had "pre-established" the future course of things in every respect and in all detail) could support the idea that, once the state of things at a time was fully known, all future states could be predicted by means of the governing laws of nature (Laplace). Which means that nothing *really new*, nothing that has never existed before, could ever happen in the course of time, a view that among others must dismiss the creative power of the "free will" of living Beings which for Newton was a reality beyond all question (cf. the Scholium generale to the Principia, book III).

As science developed after Newton on Leibnizian principles, "causality" and "determinism" became confused, so that many who speak of "causality" actually mean determinism in the sense just explained, while others who speak of "determinism" mean to use a term that would correctly imply causality. Sometimes one can even find the term "causal-deterministic". To avoid this confusion, one should use the term "causality" or "causation" to designate the "generation of the new" by an *active cause*, while "determinism" should designate a chain of necessities which connects the remotest past predictably (since nothing really new can intervene) to the remotest future.

III The Limited Power of Algebra

So is it irrevocably the fate of science to proceed with algebra and on statistics and probability theory due to algebra's limited powers to access true reality? No, it is not. Strictly speaking, in contrast to Judea Pearl's high praise of it, algebra has never been the fate of natural science until today. The limited powers of algebra *with respect to Nature*, known to Galileo 400 years ago, have again been realized in our time, for example in Gregory Bateson's 1979 book "Mind and Nature". Bateson found that *logic is an incomplete model of causality*. In the following review I shall try to go a bit beyond Bateson.

1. About fifty years after J. L. Lagrange had perfected L. Euler's algebraic theory of motion as "analytical mechanics" [27] the new science of thermodynamics [28] would emerge. In 1824 the engineer Sadi Carnot (1796-1832) published his paper "Réflexions sur la puissance motrice du feu et sur les machines propres à developper cette puissance". Considering the function of steam-engines, Carnot discovered what Galileo and Newton had already known: In contrast to logical reasoning (from reason to consequence) which is timeless, and can often be reversed (Gregory Bateson), natural processes of change (from cause to effect) always happen in space and time, and in the one-way course of time from past to future (the so-called "arrow of time"). Carnot observed that a "flow of heat" always occurs in only one direction (from high to low temperature). This was basically the second law of thermodynamics. Unfortunately, he did not analyze the subject according to Newton's laws of motion, nor did he follow Newton's advice "by the way of analysis (to) proceed from compounds to ingredients, and from motions to the forces producing them, and in general, from effects to their causes, and from particular causes to more general ones, till the argument end in the most general". Rather he considered Leibnizian "states" of a technical system, and the balance of such homogeneous states; and when speaking of the "motive force" of steam he tacitly used a compound concept of scalar "energy" in the Leibnizian sense, even though he didn't present his considerations in a mathematical form.

What emerged in the course of the 19th century through the works of Carnot and others [29] was a thermodynamic theory of "compound states of systems" that implied, but also obscured, the geometric law of cause and effect as well as its ingredients "force" and "change of motion". As a consequence, thermodynamics does not consider "effects" as phenomena of (change of) motion, but rather as "changes of state", and does not ask for the causes of such changes of state, but rather sees them "emerge" somehow. Nevertheless, the true geometric law of cause and generated change of motion as its effect was and is at work behind the curtain, as we shall see in detail below; and this guarantees the unchallenged *efficiency of applied thermodynamics.* [30]

A somewhat different development (albeit on similar grounds) happened in *hydrodynamics* in the middle of the 19th century. Theoretical fluid mechanics as conceived at that time by George Gabriel Stokes (1819-1903) *differed from reality in so many respects that engineers developed an empirically based 'technical hydraulics' of their own* [31]**.** He who studies this subject will find that these engineers introduced *constants of proportionality* into the theoretical equations, which meant to give the formulae back a geometrical structure representing proportions A/B = C = constant.

2. This case grew more obscure when in the middle of the 19th century the algebraic theory of analytical mechanics was also established as no longer one of force and (change of) motion (of cause and effect), but as a mechanics of "states" and "changes of states" according to the Hamilton-Jacobi theory. Based on Leibniz's scalar concept of "energy" (i.e. the formula E = mv^2, which Gustave Gaspard Coriolis in 1829 had enlarged arbitrarily by the factor ½ for the sake of better integration facilities [32], this formalism also (and also unnoticed) implied the true cause-effect relation (the energy-momentum proportionality, that is), as we shall see below. Note, by the way, that the factor "1/2" popping up here and there in theoretical physics without any real meaning, was added just arbitrarily, as a mathematical tool. Its omnipresence in theoretical physics, where it even determines the spin quantity of electrons, is a striking instance of the method to confuse pure mathematics with natural science, e. g. to hypostatize mathematical "singularities" as real representatives of natural entities, thus producing what some people rightly call "mathe-magics". To mathemagics belongs the "non-locality" miracle (a particle allegedly mysteriously existing at different places in space at the same time) that results from Schrödinger's wave mechanics according to the "Copenhagen interpretation". Another instance of hypostatizing happened when the concept of "mass", which in Newton's quantum theory [33] meant just the integer number of material particles that constitute a macroscopic body [34], was misunderstood and defined as a real natural property of matter to make it heavy, and inert, and to provide it with active and passive powers of attraction. (As a consequence, physicists began to search for the origin of this mysterious "mass". As we know, they recently found it: the Higgs particle).

It should also be stressed here that the Leibnizian concept of energy characterizes not some real natural entity but a *compound of* such entities like the vectors momentum p (= mv) and velocity v which, taken together, yield the scalar term mv^2, which term Leibniz erroneously thought to represent a quantity of a generating "force of motion" [35]**.** Alfred North Whitehead was right when he, in his 1929 book "Process and Reality", wrote that all basic natural quantities must be not scalar but vectorial (*directed*, as I would say, in space and time: from here to there, from now to then, from cause to effect).

Newton describes a scalar compound or "state of a system" under the name of "action" in the Principia, in the final part of the Scholium after Corollary 6 to the laws of motion. After having explained wherein "the effectiveness and usefulness of all machines and devices consist", Newton writes: "By these examples I wished only to show the wide range and the certainty of the third law of motion. For if the action of an agent is reckoned by its force and velocity jointly, and if, similarly, the reaction of a resistant is reckoned jointly by the velocities of its individual parts and the forces of resistance arising from their friction, cohesion, weight, and acceleration, the action and reaction will always be equal to each other in all examples of using devices or machines.". The gist of Newton's third law is indeed "action = reaction". The heterogeneous ingredients of the listed compounds, if properly measured, and if their measures, or dimensions, are taken together, on each side of this equation will yield compounded terms the dimensions of which mirror exactly the dimension of Leibniz's "force", or "energy", namely the scalar mv^2 (dimensions "motion (mv) times velocity(v)", that is the compound dimension [mL^2/T^2] [36].

Therefore, Newton's third law of motion represents indeed a kind of a bookkeeping principle of Nature to balance its actions by equivalent reactions, as it is also the true contents of the third law of thermodynamics. This law describes a "background of Being" of systems which it refers to "absolute temperature = zero". This state (no action, no reaction) basically establishes the (Parmenidean) persistence of Being. The fact that this scalar energy term *is a compound* also shows that it can never represent a basic physical quantity, or a "quantum", since a compound ultimately consists of at least two ingredients. The same is true, of course, with respect to "entropy" as the thermodynamic counterpart of "energy" [37].

But it is not only the third law of thermodynamics which has its counterpart in Newton's theory of motion. The same can be said of the first and second law. Actually, when thermodynamics emerged as a practical science (based on experience only) to open besides classical mechanics "a second window" to nature, scientists unconsciously developed the three laws analogously to the true contents of Newton's authentic three laws. With a grain of salt, one can say that Newton's first law refers to a natural "ground state" of motion which is "rest" (a state empirically indistinguishable from "uniform motion", which nevertheless can be measured and so identified geometrically); and this is analogous to the first law of thermodynamics understood as referring to the category of Being that appears locally as "a system". The second law refers to natural "transformations of the ground state" for which Newton uses the term "change in motion", and to the natural cause thereof called "impressed motive force" (*vis motrix impressa* in Newton's Latin); and this is analogous to the second law of thermodynamics referring to the transformation properties of systems. Note that the "time arrow" that characterizes the second law of thermodynamics is missing only in the algebraic textbook misrepresentations of Newton's second law, but is present in the geometric representation of that law as a quaternary proportion. The analogy between Newton's third law and the third law of thermodynamics has already been explained above [38].

3. In 1864, James Clerk Maxwell (1831-1879) published his mathematical theory of electromagnetism which in a way re-introduced geometric considerations into the algebraic formalism. It also implied the geometric cause-effect proportionality: In 1884 it was Henry Poynting (1852-1914) who from the Maxwell equations extracted the formula that was baptized "Poynting vector" mentioned above, exhibiting (a flux of) energy E for the first time as a *vector quantity*. Basically it can be written in the form $E/p = c$ = "vacuum velocity of light", and to this day this formula provides a part of the accepted theory of radiation pressure. Note that the proportionality factor c (geometric dimensions "space over time") only in the Maxwell theory became identified with the at that time just-discovered, apparently constant, "vacuum velocity of light".

4. In 1900, Max Planck (1859-1947) was forced by experimental evidence to publish his formula $E = h\nu$, equivalent to $E \propto \nu$, or $E/\nu = h$ = constant, which is a geometric proportionality between (vectorial!) "energy", E, and (also vectorial) radiation frequency, ν. This concept of energy should show energy being "quantized", to really exist in discrete portions or "particles", and the formula was meant (and still is) to also show the most elementary "quantum of action" being just h, "Planck's constant". It can easily be shown, however, that Planck's formula also implies the proportionality of energy and momentum to result in the constant c. Recall that some years later the constant h was found to be "h = momentum p times wavelength λ" (derived from de Broglie's formula $p = h/\lambda$). With $\lambda\nu = c$ there results $p = h\nu/\lambda\nu = E/c$, equivalent to $E/p = c$ = constant. No wonder, then, that Planck's formula became the foundation of the new Quantum Mechanics (QM), into the formalism of which, however tacitly and

unnoticed, this formula introduced the well-known realist geometric law of cause and effect, basically E/p = c = constant, to guarantee the rightly praised efficiency of QM. [39]

5. In 1905 Albert Einstein (1879-1955) published a paper that basically contained what later was called “the theory of special relativity”. In a second paper published also in 1905 he brought to light the formula E = mc^2 as a consequence of the first. One year later Einstein published a third paper in which for the first time appears his assertion of an “equivalence” of mass (or matter) and energy. In 2004 I have publicly demonstrated mathematically (using among others an argument formulated by Max Jammer) this to be a momentous misinterpretation [40]**.** Note that the alleged mass-energy equivalence plays an important role in particle physics, so that its long overdue correction might mean a serious impact on the prevailing standard model, and on the interpretation of the “Higgs particle”. Basically the argument is that an “equivalence” of mass and energy would read E = m which is not Einstein’s equation, which equation correctly must be read as a *geometric proportionality of energy and momentum* again, according to $E = p \times c$ **(** with p = mc representing the “momentum of radiation”, the formula reads $E = mc \times c$, or E = mc^2, the Einstein equation). [41]

6. In 1918 Emmy Noether (1888-1935) conceived what was later called the “Noether theorem”, the consistence of which has never been called into question. The theorem exhibits a symmetric quaternary mathematical compound, composed of four natural entities “energy E, time t, momentum p, and space s” that can be written $\Delta E \times \Delta t = \Delta p \times \Delta s = h$ (Planck’s constant), thus showing an equation of products. Now, if we transform it into an equation of quotients according to a well-known geometrical rule, we obtain $\Delta E/\Delta p = \Delta s/\Delta t$ = constant, which is again our formula $\Delta E/\Delta p = c$ = constant, or generally: E/p = c. Therefore the Noether theorem corroborates the existence and validity of a geometric natural law “energy over momentum = c = constant”, showing energy E as a vector quantity in a linear relation to momentum p. The theorem also corroborates what has been said in footnote 37 of the QM vectorial energy term to differ from that of classical mechanics, the scalar E = mv^2/2 (= p^2/2m as it is mostly used in QM).

7. Seven years after the formulation of the Noether theorem, in 1925/6, both Werner Heisenberg and Erwin Schrödinger conceived their mathematical theories of QM, using very different methodical approaches to the subject. They asserted these approaches to be equivalent, and this is generally believed today. But this equivalence claim is not true, even though the

great mathematician John von Neumann (1903-1957), meant to have demonstrated it. [42] Actually Erwin Schrödinger based his theory on the Leibnizian scalar concept of energy only (in the form E = p^2/2m). Werner Heisenberg, on the other hand, used the vectorial concept $E = p \times c$ [43], the geometric proportionality of E and p. As a result of the basic energy concept, in Schrödinger's theory there appears the term $E = p \times v/2$ (to result in E = p^2/2m) showing the factor v/2, a variable, at the very place where Heisenberg has the constant, c. Von Neumann was not aware of this difference (and nobody since has been, so far as I know), because he did not consider the geometric contents of Heisenberg's theory, and consequently did not distinguish the variable v/2 and the constant c (both terms being apparently identical on grounds of dimensions).

This contents comes again to light when the Noether theorem is applied to the Heisenberg relations, reading $\Delta E \times \Delta t \geq h$; $\Delta p \times \Delta s \geq h$. Put together according to $\Delta E \times \Delta t = \Delta p \times \Delta s$ (which formula Niels Bohr introduced to the fifth Solvay Conference in 1927; note that h vanishes), and this equation of products transformed to $\Delta E/\Delta p = \Delta s/\Delta t = c$, we obtain once more the geometric law of causality, being an integral part of Heisenberg's QM. It should well be noted here that this geometric analysis removes every "uncertainty" from this theory. Moreover it explains the so far unexplained fact that some quantum mechanical operators do not commute, which means that their proper places in the equations cannot be changed arbitrarily without doing damage to the results of their application. The explanation of this non-commutativity is that exactly *this order* of terms in a quaternary geometric proportion, and exactly *this relation* of terms to each other, *namely the proportionality of energy and momentum, and of space and time*, is the real one, that exactly *speaks the language of Nature*. The latter proportionality of space and time, by the way, becomes apparent to anyone who reads Galileo's "Two New Sciences", and looks at the figure to prop. 1 theor. 1 in the chapter "Third Day" (On local motion).

V Geometry Hidden – Geometry Uncovered

In the year 1959, the Physicist Eugene Wigner (1902-1995) delivered a Richard Courant Lecture at New York University, entitled "The Unreasonable Effectiveness of Mathematics in the Natural Sciences [44]. His subject, insofar as "mathematics" is concerned, was algebra, of course. What he didn't realize is the fact that "mathematics" is only effective in natural science, and has always been, when the "machinery" of *geometric* principles works at least in the background. Nature, all in all, speaks not algebra but geometry – Euclidean geometry that is. Galileo was right. Newton also was right on this. As a young man he had become enthusiastic about the then new algebraic mathematics of Descartes. It is known that he nearly failed the 1664 Cambridge election to scholarship, because he, when he was sent to Isaac Barrow to be examined, and was found a master of Descartes' analytic geometry, was also found to know little or nothing of Euclid. In his later years he more than once uttered his regret for not having earlier studied instead of Descartes the geometry of the Ancients.

It must be said here, however, that already some of those early scientists who unwaveringly believed in the Cartesian-Leibnizian algebra nevertheless felt its various shortcomings when applied to Nature. When they became aware of algebra's restricted powers, and resorted partly to probability theory, partly to an extension of theoretical algebra, geometry was always tacitly present, or reappeared in the background.

1. Some thought already in the 18th century of probability theory to provide a better tool. One of them was Thomas Bayes (1701-1761) whose work plays a special role in Pearl's book. On p. 5, Pearl introduces a formula 1.13, which, as he says it, "states that the belief we accord a hypothesis H upon obtaining evidence e can be computed by multiplying our previous belief P (H) by the likelihood P (e | H) that e will materialize if H is true". However, this formula, which he calls "The heart of Bayesian inference", shows the reader something that is clearly a geometric quaternary proportion, written $P\ (H \mid \mathrm{e}) : P\ (e \mid H) = P\ (H) : P\ (e)$. As one can see, the geometric tetraktys exhibits its power even at the core of the allegedly throughout algebraic work of Thomas Bayes to calculate with precision degrees of belief, or of probability.

2. In 2004, Robert Penrose published his book "The Road to Reality" which – according to the subtitle – provides "A Complete Guide to the Laws of the Universe". [45] In his preface the author says that his book is "really about the relation between mathematics and physics." Actually he could have said: about *arithmetic and algebra* and physics, since the main contents of the book is mathematics as identified with algebra. Especially in the first twelve chapters of the book the author gives a survey of the difficulties in developing a mathematics that might fit with reality. But in chapter 12 "Manifolds of *n* Dimensions" there appears on p. 238 the symbol " $\propto$ " to denote "being proportional to". The author, however, neither in this place nor in his chapter on "Notation" (pp. XXVI-XXVIII) bothers to explain the meaning of the symbol, let alone the power of geometric proportion theory (to which it belongs), rather he

introduces the symbol casually, as a kind of algebraic shorthand “so that our expressions do not get confusingly cluttered with complicated-looking factorials” (p. 238).

The resulting formulae Penrose calls “part of classical tensor algebra.” This may well be true. Nevertheless it shows that the author doesn’t know anything of geometric proportion theory, as it is presented in Euclid, Elements, Book V, definitions 1-6. The case moreover reveals that “algebra” when applied to reality, *ultimately must become so expanded as to be able to work with “proportional” natural entities of a different kind*. “Proportionality signs $\propto$” consequently must, and do, appear when the author on p. 242 explains how “symmetric and anti-symmetric parts of general tensors can be expressed”. In chapter 13 “On Symmetry Groups” the author develops formulae that lead to the notion of “commutativity” (p. 249, 260), to end up on p. 293-6 with “non-commuting variables”. The “non-commutativity” of heterogeneous variables, or incommensurables, when put in geometric quaternary proportions, is a basic characteristic of geometric proportion theory, as hase been noted above.

On p. 251 Penrose muses “that there is some fundamental symmetry of nature that relates different kinds of particles to one another and also relates different particle interactions to one another”. The author seems to have felt the need of a mathematics to describe such a symmetry, but evidently is not aware of the facts that (1) a relation of *different kinds* of particles is fundamentally *asymmetric*, or “broken” (cf. Newton, Principia, Scholium after Lemma X to the laws of motion), and, (2) that such a mathematics *is already at hand since the time of Euclid*, with the geometric theory of proportions (which theory the author nowhere in his voluminous book of 1099 pages mentions).

On p. 259 and again on p. 285 there appears the formula $\boldsymbol{TT}^{-1} = \boldsymbol{I} = \boldsymbol{T}^{-1}\boldsymbol{T}$ which is nothing but a quaternary geometric proportion between variable quantities of a different kind, the middle $\boldsymbol{I}$ playing the role of the factor of proportionality**.** Proceeding with the author to “13.6 Representation theory and Lie algebras”, “13.7 Tensor representation spaces; reducibility”, “13.8 Orthogonal groups”, and “13.10 Symplectic groups”, the reader learns what an increasing role the increasing number of “signatures” plays here. *Actually they serve to distinguish quantities of different kinds from each other, and teach how to connect such quantities with each other,* and to this difference and connection problem points the notion “symplectic” (oddly and unintelligibly defined on p. 285). Therefore, in “15.7 Non-triviality in a bundle connection” on p. 346/7 we read of the “extension of the connection from vectors to different kinds of entity” (extension from mathematics to physics, that is), and can learn on p. 347 the structure of a “bundle connection” (which is basically the structure of a geometric proportionality A/B = C = constant). No wonder, then, that all this, when applied to physics, leads the author to present (in chapter 20 “Lagrangians and Hamiltonians”) a result that combines “bundle connections” and the “symplectic structure” of “symplectic manifolds” (p. 476), culminating in “20.4 Hamiltonian dynamics and symplectic geometry” in an allegedly new geometry with most powerful features.

What is “symplectic geometry”? What is “symplectic”? The Greek notion “plektos” means “woven”, consequently the notion “symplectic” will mean “tightly woven together from two different threads” (so as *warp* and *weft* in weaving) . And this is nothing else but “geometric proportionality” as explained in Plato’s Timaios, and in Euclid’s Elements. “Symplectic geometry” as we find it in Penrose’s book therefore can be understood as “geometric proportion theory algebraically disguised”, or “algebraic geometry”, which is actually the name of a branch of modern theoretical mathematics.

On p. 485/6 Penrose writes: “Nature has had a habit in the past of first tempting us to a euphoric complacency by the power and elegance of the mathematical structures that she appears to force us to accept as guiding her world, but then jolting us, from time to time, out of our conceptual torpor by showing us that our picture could not have been correct, after all!”

Indeed, that is it. The “euphoric complacency” is what the “Geometers” felt when they discovered the power of Euclidean geometry as a tool to uncover the secrets of Nature. “Nature and Nature’s laws lay hid in night, God said: Let Newton be! – and all was light.” So said Newton’s contemporary Alexander Pope. The wrong way was taken when scientists of the 18th century departed from geometric proportion theory, trying to understand Nature by means of algebra, or to subjugate it to the logical laws that govern the human mind – but not Nature. Nature herself, however, as the instance shows, tends to lead science, when it got lost, back to her reality and truth, even by “jolting” the algebraist so that he unintentionally must return to Euclidean geometry.

3. Another striking example of starting with algebra rightly understood as an insufficient tool of natural science, ending up with a kind of geometric proportion theory, gives the biologist Robert Rosen (1934-1998). In his book “Life Itself” [46] he starts with a harsh and well-ground-

ed criticism of algebraic classical mechanics, which he somehow identifies with “the machine metaphor” that “is not just a little bit wrong; it is entirely wrong and must be discarded” (p. 23). Of course Rosen (as well as nearly every body in the world so far) understands the criticized mechanics as “Newtonian” (see his explication of “Newton’s laws” p. 93/94), erroneously identifying Newton’s second law (in consequence of the mistaken formula $F = mx$” on p. 94, tantamount to F = ma) as a “recursion rule” (i.e. as a time-symmetric law) , and basing all that on “The Concept of State” (chapter 4). The consequence he draws from the evident shortcomings of applying classical mechanics to Nature herself is to raise a “Back to the Basis” claim (in chapter 3A), and in consequently proposing a theory “to leave the world of mechanism without giving up science” (p. 243). This theory Rosen calls “relational theory”, presenting its basics in chapter 9 “Relational Theory of Machines”. And, as the reader may already have presumed, this theory as well is (unwittingly, however) established on the concept of geometric proportionality of quantities of a different kind. To prove this, on p. 221 Rosen gives a formula $f : A \to B$ which he calls “a *ternary relation* between *f*, *A*, and *B*”.

As we see, once again Nature has led a scientist “back to the basis”, i.e. back to geometric proportion theory (here again appearing as the “rule of three”).

4. The same is the case with Adrian Bejan’s and J. Peder Zane’s book “Design in Nature” which dates from 2012 [47]. The book presents an ostensibly new law of Nature called “the constructal law”, which – according to the subtitle – “governs evolution in biology, physics, technology, and social organisation”. I do not go here into details of this claim. Rather I want to point to the fact that these authors at least are aware of the central role which geometry (Euclidean geometry) plays in explanations how Nature really works. “Design phenomena are not covered by the existing laws of physics” the authors state on p. 18; and this is well observed insofar as one refers to the present laws of physics that are throughout algebraic and consequently do not speak the language of Nature. “Rivers follow geometric rules predicted by the constructal law” is what they say on p. 25. And the first part of this assertion is certainly true (except of an unclear use of the term “prediction”). On p.189 the authors (without giving a geometric explanation, though) introduce a quaternary geometric proportion $H/L = 2V_o/V_1$ in a context that is meant to describe the optimization of passenger travel through an airport’s area. Another quaternary proportion to read “$L/H = V_L/V_H$ “ appears on p. 226. This formula the authors present in chapter 9 under the heading “The Golden Ratio, Vision, Cognition, and Culture”. Here they refer explicitly to Euclid’s Elements, and to the theory of geometric pro-

portions, even calling “the ‘mystical’, timeless secret to the golden ratio” (if there should be something like that) “the fact that it connects humanity to nature”. A propitious phrase; Plato, Galileo, and Newton would agree.

5. Judea Pearl himself is one of the promoters of algebra who have tried to improve this mathematical tool so as to be able to grasp true causal relations. He comes to understand that a solid causal relation requires at least three terms (cf. the inductive causation algorithm – IC algorithm, p. 50). E.g. on p. 54 under the heading “Local Criteria for Inferring Causal Relations” he introduces “explicit definitions of potential and genuine causation as they emerge from the IC algorithm” developed by himself, and then notes “that, in all these definitions, the criterion for causation between two variables (X, and Y) will require that a third variable Z exhibit a specific pattern of dependency with X and Y. This is not surprising, since the essence of causal claims is to stipulate the behaviour of X and Y under the influence of a third variable.” Which is certainly true, aside from taking the third term as also a variable. One of the three terms must be a special one, even though the “Borromean Rings” show that anyone of the three rings can play this role. With respect to mathematics, however, geometry tells us that the “third ring” must be an *invariant*, a *constant*, resulting from the relation of the first two ones to each other, able to work as a factor of proportionality generating the causal “specific pattern of dependency with X and Y” (Pearl) which I call “geometric proportionality”. On p. 54 we also read: “The IC* algorithm can be regarded as offering a systematic way of searching for variables Z that qualify as virtual controls, given the assumption of stability.” This reference to “stability” seems to somehow point in the direction of *invariance of the third term*; but only “somehow”. And, the general shortcoming of Pearl’s IC algorithm is to ignore the *heterogeneity* of the three terms, which to describe algebra is not able on principle, so long as it is based on the logical axiom of non-contradiction, A (= B) = A.

VI The Language of Nature is Geometry

Galileo Galilei in his 1638 “Discorsi” among others presents the following striking example to show that the language of nature is not logic, or algebra, but Euclidean geometry. Take a piece of cloth that is twice as long as it is wide. Sew it lengthwise to form a sack for holding a certain quantity of grain. Take a same piece of cloth and sew it the other way (rolling it around the wide side). Now you have two sacks, one narrow and high, one wide and low, both made of the same quantity of cloth. Will both sacks also hold the same quantity of grain? Logic says yes; geometry demonstrates the opposite, and experience corroborates it: No, the wide and low sack will actually hold twenty-five times the quantity of which the narrow and high sack holds just seven. Evidently it is a certain property of space that plays a decisive role here, a property which logic and algebra alone would never have discovered [48]. There is another example in the "Discorsi" to demonstrate the priority of geometry. Is it possible that an object moving in an accelerated manner could describe spaces (distances) in proportion to the increasing velocity, so that the body when having doubled its velocity would also have doubled the space described, according to a proportionality of velocity and space? Logic says that this must of course be possible. But Galileo demonstrates geometrically that this is absurd, because in this case the moving body would mysteriously occupy different places in space at the same time [49]. A third example is also related to the absurdity of velocity-space proportionality. It was G. W. Leibniz who in the year 1686 constructed a quantitative measure of "force of motion" by assuming that this force should be proportional to the space described by a body ascending vertically against gravity. From this hypothesis resulted a concept of force that was measured not proportional to motion (momentum), but equal to the square of velocity. Leibniz's mistaken account was rightly called "a wonderfully philosophical error" by Isaac Newton, and strongly criticized by Samuel Clarke (see footnote 19). Nevertheless after Newton's death it became a most basic concept of algebraic classical mechanics under the name "energy" (pesumably first used by Thomas Young in 1807). This could only happen because the algebraic formalism obscures the absurdity of the concept which can only be seen with the help of geometry.

These examples and demonstrations of the power of geometry should suffice, and allow us to say for true: The language of nature is geometry – Euclidean geometry, that is.

This result does not mean that science must return to Galileo and Newton, but that scientists should be aware of the true geometric background and contents of the basic modern developments, relativity and quantum mechanics. It is a fact that special relativity and the Heisenberg QM both imply a constant quantity which is a quotient of an element of space and an element of time, showing the geometric proportionality of space and time, of energy and momentum, of cause and effect. It doesn't matter that this constant has been baptized "vacuum velocity of light". The only important thing is that in relativity and in Heisenberg's quantum mechanics it serves as a cosmic space-time *frame of reference and measurement of material motion* *relative to it.* So "motion" and its true cause seems to be topic that should again be considered in natural science, instead of "energy states".

As the said theories of motion imply this reference frame I call them "cosmocentric", in contrast to others which refer not to the geometry of space and time but to the logic of human reasoning; these I call "anthropocentric". To the latter category belongs non-Newtonian non-geometric "classical mechanics" (the algebraic theory of Leibniz and the Leibnizians) and the Schrödinger formalism of QM (the wave mechanics). Insofar as the cosmocentric theories are rooted in the objective cosmic reality of space and time I call them "real" and "true", while the anthropocentric theories, making man's logical reasoning the only measure of things, lack a true relation to the objective reality and truth of nature. Therefore to deny that natural science has to do with truth would mean to deny that it has to do with the reality of nature.

To distinguish between cosmocentric theories (such as the Copernican system) and anthropocentric theories (such as the Ptolemaic system) seems to be extremely helpful in distinguishing what is real or true from what is rational and hypothetic [50].

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Judea Pearl has in his book a saying of Einstein (1953) as a motto on p. V to read: "Development of Western Science is based on two great achievements: the invention of the formal logical system (in Euclidean geometry) by the Greek philosophers, and the discovery of the possibility to find out causal relationships by systematic experiment (during the Renaissance)."

This may be correctly quoted, but implies two big errors. One is, that "the formal logical system" cannot be found "in Euclidean geometry", since geometry basically is not a *logical* but rather an *onto-logical* system. Geometry is not a free invention of the human brain, as some believe; therefore it does not follow the same rules as human logical reasoning. Being "founded on mechanical practice" (Newton), geometry rather mirrors an intrinsic structure of Nature herself, as has been demonstrated above.

The second error is that Einstein, speaking of "the discovery of the possibility to find out causal relationships by experiment (during the Renaissance)", tacitly imposes a "positivistic" note upon Renaissance science, as if it would have been established on experiment, and on observation of phenomena only, and, as if causal relationships would have been understood from the beginning of modern science as relationships *among phenomena* (events).

In contrast, Renaissance science, as it culminated in the natural philosophy of Galileo and Newton, was erected by means of “reason and experience” (ratio et experientia) combined; and, note that this “reason” was identical with the knowledge of the powers of Euclidean geometry, and of geometric proportion theory (i.e. the tetraktys) to discover and identify by measurement the true unobservable and nonmaterial causes of the observable natural phenomena.

Pearl should better have taken another saying of Einstein. It reads: *Insofar as the propositions of mathematics refer to reality they are not solid, and insofar as they are solid they do not refer to reality.* This saying requires a short comment as well as the other one. The comment here is that Einstein is certainly true ***with respect to the propositions of algebra*** when applied to reality, or Nature, ***because, as we have seen above, the language of Nature is not algebra.***

Ed Dellian.

Footnotes and references.

1 Judea Pearl, Causality – Models, Reasoning, Inference, Cambridge Univ. Press, 2000, 2nd ed. 2009.

2 Galileo Galilei, Discorsi e dimostrazioni matematiche intorno a due nuove science attinenti alla mecanica ed i movimenti locali (Elsevier, 1638); transl. Two New Sciences Including Centers of Gravity and Force of Percussion, Stillman Drake ed., Wall & Emerson, Inc., 2nd ed. 2008.

3 I refer to Shmuel Sambursky, Der Weg der Physik, Artemis, 1975, 29; 35. Sambursky correctly notes that only in the course of the 18th century natural science became based on "differential equations instead of ponderous geometrical methods", tacitly and mistakenly assuming that this would have meant nothing but a "technical innovation". This error to assume that algebra would only mean an improvement without substantial alterations of the former geometric method, also occurs in Arthur von Oettingen's German edition of Galileo's Discorsi (see Galilei, Unterredungen und mathematische Demonstrationen … Discorsi), Wissenschaftliche Buchgesellschaft, 1973, in the editor's notes, p. 131-3).

4 See Galileo Galilei, Il Saggiatore nel quale con bilancia esquisita e giusta si ponderano le cose contenute nella libra astronomica e filosofica di Lotario Sarsi sigensano scritto in forma di lettera, Roma 1623, p. 232: "La filosofia è scritta in questo grandissimo libro che continuamente chi sta aperto innanzi a gli occhi (io dico l'universo), ma non si può intendere se prima non s'impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro laberinto." *Philosophy is written in this wonderful book that stands continuously before our eyes (I mean the universe), but one cannot understand it without having first learned to understand the language in which it is written. It is written in mathematical language, and the characters are triangles, circles, and other geometrical figures, without which means it is impossible to understand even a single word; without which one hopelessly will wander around in an obscure labyrinth* (author's transl.).

5 Isaac Newton, Philosophiae naturalis principia mathematica, London 1687, newly transl. and edited by I. Bernard Cohen and Anne Whitman, as "Isaac Newton, The Principia, Mathematical Principles of Natural Philosophy", Berkeley and Los Angeles, 1999. I quote from the "Author's Preface to the Reader", dated "Trinity College Cambridge, 8

May 1686" what follows: "*Geometry* postulates that a beginner has learned to describe lines and circles exactly before he approaches the threshold of *geometry*, and then it teaches how problems are solved by these operations…. And *geometry* can boast that with so few principles … it can do so much. Therefore *geometry* is founded on mechanical practice and is nothing other than that part of *universal mechanics* which reduces the art of measuring to exact propositions and demonstrations." (p. 382).

6 The "Geometers" was a name for those early scientists (philosophers, as they saw themselves; the name "scientist" was not yet born) who, perhaps beginning with Nicolaus Cusanus (1401-1464), re-invented the Platonic natural philosophy based on geometry, and made Euclidean geometry in the Renaissance the proper tool of art (Leonardo; Dürer) and philosophy. The list includes famous names as Marsilio Ficino (1433-1499, who is said to have refounded the "Platonic Academy" in Florence); Copernicus (1473-1543); Giordano Bruno (1548-1600, burnt in Rome at the stake for heresy); Johannes Kepler (1571-1630); Galileo Galilei (1564-1642; 1632 in Rome sentenced to lifelong house arrest for heresy); Isaac Newton (1642-1727). However, some other mathematicians, e.g. Gottfried Wilhelm Leibniz (1646-1716), also called themselves sometimes "geometers", lumping together the name of their favourite tool algebra with that of geometry to just "mathematics", and then misusing for themselves the acknowledged name "geometer" as a synonym of "excellent mathematician."

7 Here I refer to "La géométrie" of Descartes, Elsevier, 1637.

8 Newton's *forces* or *active principles* of Nature, are nonmaterial natural entities, the sources of which are really existing nonmaterial "fields" as we would say. One such thing is the gravitational field described by Newton in the definitions 5 – 8 to his 1687 Principia. See also the next footnote (Chomsky).

9 In an essay "Language and Nature" (Mind, vol. 104, 413, Jan. 1995), the linguist Noam Chomsky reflects on the relations of ordinary language to nature, but doesn't care whether nature speaks algebra or geometry. Nevertheless he characteristically begins with some historical remarks concerning *causality* in the context of interaction. Remarkably Chomsky, after at first recalling the Cartesian materialist view of the world as a machine, and of interaction as only possible through direct contact of material objects, correctly introduces Newton's opposite "anti-materialist" picture of the world, relying "heavily on spiritual forces". But then, confusing *gravity* (which is an observable effect called "weight") with *gravitation* (which is the cause) he asserts that Newton, allegedly not having known the cause of gravity, had himself "regarded the principle he postulated as an absurdity". Citing exclusively the historian of science E. J. Dijksterhuis, Chomsky comes to speak of Newton's "mysterious force", mysterious as it admitted interaction without direct contact. In the end Chomsky freely asserts that Newton's "I frame no hypotheses" would have been "an expression of his concern over his inability to assign the cause of gravity".

10 Isaac Newton, Principia, p. 382: "The whole difficulty of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces." In the "Opticks" of 1717 which I find in "Isaac Newton opera quae exstant omnia", Samuel Horsley ed., London 1779-1785" in vol. IV as "Optics: or a treatise of the reflections, refractions, inflections and colours of light", on p. 263 Newton, when rejecting the method of deduction from hypotheses, explains his own method, writing: "By this way of analysis [analyzing experiments and observations; ED] we may proceed from compounds to ingredients; and from motions to the forces producing them; and in general, from effects to their causes".

11 Cf. Newton, Principia, Scholium after Lemma X to the laws of motion: "If indeterminate quantities of different kinds are compared with one another and any one of them is said to be directly or inversely as any other, the meaning is that the first one is increased or decreased in the same ratio as the second or as its reciprocal. And if any one of them is said to be as two or more others, directly or inversely, the meaning is that the first is increased or decreased in a ratio that is compounded of the ratios in which the others, or the reciprocals of the others, are increased or decreased. For example, if A is said to be as B directly and C directly and D inversely, the meaning is that A is increased or decreased in the same ratio as $B \times C \times 1/D$, that is, that A and BC/D are to each other in a given ratio."

12 See Euclid, Elements, Book V, Definitions 1-6. The said characteristic of "proportions" is explained in all detail in John Wallis, Mechanica sive de motu tractatus geometricus, London 1669-1671. Note that John Wallis is one of the very few of Newton's contemporaries to be cited and appreciated in the Principia.

13 Cf. Ed Dellian, Newton on Mass and Force, Physics Essays vol. 16 (2003) nr. 2 (also to be found on my website, www.neutonus-reformatus.com, entry nr. 18).

14 Gregory Bateson, in his 1979 book "Mind and Nature. A Necessary Unity" correctly notes that *causality implies time, while the 'if – then' of logic is timeless* (I refer to the German edition of Bateson's Book: Geist und Natur. Eine notwendige Einheit; Frankfurt am Main 1982). Cf. also Walter Elsasser, A Form of Logic Suited for Biology, in: Progress in Theoretical Biology, R. Rosen ed., vol. 6 p. 23-62, 1981.

Elsasser writes on p. 2: “Classes in which the elements are individually different from each other will be designed as *heterogeneous classes.* Thus, the description of biological states and processes is carried through in terms of heterogeneous classes. Ordinary logic pays little attention to the heterogeneity of classes.” This insight returns in Robert E. Ulanowicz’s book “A Third Window”, Templeton Foundation Press, 2009. On p. 51 the author refers to “Elsasser’s result that the notion of mechanism in biology is devoid of logical underpinnings”, and asks for “a more appropriate term for the interaction of two heterogeneous classes”. To be sure, the inapplicability of classical mechanics (based on algebra and logic) to describe the interaction of heterogeneous entities (classes) is a key insight, with far-reaching consequences.

15 Terrence W. Deacon, in his book “Incomplete Nature”, W.W. Norton & Company, 2012, on p. 7 rightly criticizes the “persistent dualism” that stems from Descartes, calling it on p. 544 “the Cartesian wound that severed mind from body at the birth of modern science”. Unfortunately this author as so many others has not seen that an alternative natural philosophy was conceived soon after Descartes, which introduced a very different world view. Based on reason and experience, it taught the reasonable, geometrically describable interference of spiritual “active agents”, the nonmaterial “forces of Nature”, with the material world, according to the law of proportionality of cause and effect (Newton’s second law). For the nonmateriality of Newton’s concept of force see footnote **9.** Unfortunately, again, Deacon also believes in the *f = ma* misinterpretation of Newton’s second law(apparently the only *equation* in his volume-nous book, see p. 109), which arbitrarily obscures the nonmateriality of force, or cause in Newton’s theory of motion (cf. the following footnotes **17, 18**).

16 See Leibniz, Principium mechanicae universum novum, The Hannover Library 35, 10, 5 pp. 1-4, unpublished. This I take from Herbert Breger, „Elastizität als Strukturprinzip der Materie bei Leibniz“, in: studia leibnitiana Sonderheft 13 “Leibniz’ Dynamica”, Albert Heinekamp ed., Stuttgart 1984, p. 117/8. Breger refers to Leibniz’s „reception of Wallis“ and writes the following (p. 118; my transl.): *Wallis had noticed in his book* [cf. footnote12] *that causes were proportional to their effects which would allow for a mathematical treatment of physics. Leibniz excerpted this remark of Wallis and shortly afterward he proposed the principle that causes should be* [not proportional but] *equal to their effects*.” (my ital.). Ernst Cassirer, Leibniz’ System in seinen wissenschaftlichen Grundlagen, Georg Olms Verlag, 1980)on p. 310 writes about “Leibniz’s monism”, judging it as e*merged from the logic of science and able to refer to the problem of unity* *of experience. Accordingly a basic law can be put which unites all what happens so that the former state now can be connected with the future one by means of a mathematical* ***equation*** [emphasis in the original], *in which the quantity of 'effect' appears as an unequivocal function of the quantity of 'cause'. Cause and effect then should be two events in the course of time, lawfully related to each other. This basic law now requires the both terms to be not only proportional to each other but equivalent.* Cassirer quotes Leibniz:

"Au lieu du Principe Cartésien on pourrait établir une autre loi de la nature que je tiens la plus universelle et la plus inviolable, savoir **qu'il y a toujours une parfaite equation entre la cause pleine et l'effet entier** [original emphasis]. Elle ne dite pas seulement que les effets sont proportionels aux causes: mais de plus que chaque effet entier est équivalent à sa cause."

17 See the concept of "dead force" in Leibniz's "Specimen Dynamicum" of 1695, Leibniz's reply to Newton's 1687 "Principia". This was afterwards put into effect, as the most basic law of "classical" continuum mechanics, by the algebraist Leonhard Euler, who in the year 1750 published his version of it in Berlin, calling the law his own "discovery".

. **18** Leonhard Euler, Découverte d'un nouveau principe de Mécanique. This is the title of a paper presented to The Prussian Academy of Science, Berlin, the 3rd of September, 1750 (date according to C.G. J. Jacobi).

19 Cf. "A Letter from the Rev. Dr. Samuel Clarke to Mr. Benjamin Hoadly F.R.S. occasion'd by the present controversy among Mathematicians, concerning the proportion of Velocity and Force in Bodies in Motion,", Phil. Trans. Vol. 35 (1727-8) p. 381:

"Sir, it has often been observed in general, that *Learning* does not give Men *understanding;* and that the absurdest Things in the World have been asserted and maintained, by Persons whose Education and Studies should seem to have furnish'd them with the greatest Extent of Science. That Knowledge in many Languages and Terms of Art, and in the History of *Opinions* and *Romantic Hypotheses* of Philosophers, should sometimes be of no Effect in correcting Men's judgement is not so need to be wondered at. But that in *Mathematicks* themselves, which are a *real science*, and founded in the *necessary Nature of Things*; men of very great Abilities in *abstract* Computations, when they come to *apply* those Computations to the *Nature of Things*, should persist in maintaining the most *palpable absurdities*, and in refusing to see some of the most evident and obvious Truths; is very strange. An extraordinary instant of this, we have had of late Years in very eminent Mathematicians, *Mr. Leibnitz, Mr. Herman, Mr. s'Grave-*

sande, and Mr. Bernoulli; who (in order to raise a Dust of Opposition against Sir Isaac Newton's Philosophy; the Glory of which is *the Application of abstract Mathematicks* to the *real Phaenomena of Nature*) have for some Years insisted with great Eagerness, upon a Principle which subverts all Science, and which may easily be made appear (even to an ordinary Capacity) to be contrary to the *necessary and essential Nature of Things.* What they contend for, is, that the *Force* of any *Body in Motion*, is proportional not to its *Velocity*, but to the *Square of its Velocity….*".

20 Jean d'Alembert, Traité de Dynamique, dans lequel les loix de'l équilibre et du mouvemont des corps sont réduites aux plus petits nombre possible, Paris 1743, 2nd ed. 1758, Préliminaire, p. XI: "Nous verrons bientôt comment on peut determiner les effets de'l impulsion, & des causes qui peuvent s'y rapperter: pour nous en tenir à celles de la second espece, il est clair que lorsq'il est question des effets produits par de telles causes, ces effets doivent toujours ètre données indépendamment de la connaissance de la cause, puisq'ils ne peuvent en être deduits: C'est ainsi que sans connoître la cause de la pesanteur, nous apprenons par l'experience que les espaces décrits par un Corps qui tombe, sont entr'eux comme les quarrés des temps… Pourquoi donc aurions-nous recours à ce principe dont tout le monde fait usage aujourd'hui, que la force accélératrice ou retardatrice est proportionelle à l' element de la vitesse? Principe appuyé sur cet unique axiome vague et obscure, que l'effet est proportionel à sa cause. Nous n'examinerons point si ce principe est de vérité necessaire; nous avouerons seulement que les preuves qu'on en a apportées jusq'ici, ne nous paroissent pas hors d'atteinte: Nous ne l'adopterons pas non plus, avec quelques Géometres, comme de verité purement contingente … nous nous contenderons d'observer, que vrai ou douteux, clair ou obscur, il est inutile à la Méchanique, & que par conséquent il doit en être banni."

Here comes to light the, say, positivistic attitude of a scientist who was one of the most influential promoters of the so-called Enlightenment. I leave it to the reader's consideration whether or not it seems reasonable to "ban" a not yet fully understood principle from science (as if to punish it for the scientist's stupidity).

21 I quote Russell following Pearl, p. 408.

22 Cf. footnote 9 **(**Noam Chomsky). Recently I read Karl Popper's "The World of Parmenides – Essays on the Presocratic Enlightenment", Routledge, 1998, to find the following: "Evidently Newton did not believe in 'reversibility' in spite of the evident reversibility of Newtonian Dynamics." Never did Popper ask himself whether this apparent contradiction might perhaps be explained through an error of translation from the Latin, or other interpretation. Such an error, however, is evident for him who reads Newton's second law in Newton's original Latin version, where there is no mentioning of a *reversibility* (equality of cause and effect, that is).

23 The fact that Newton held matter to be absolutely passive, unable to act by itself, which is the contents of the first law of motion, should have hindered scientists to believe that according to Newton "the earth" could attract bodies at a distance. Nevertheless this belief is present in every modern textbook of classical mechanics.

24 Principia, the Cohen-Whitman translation, p. 295 (Cohen's introductory "Guide to Newton's Principia").

25 One great scientist of the Enlightenment was Etienne de Condillac (1715-1780). In his "Traitè des systèmes" (1749) he explicitly claimed that in science geometry should be dismissed, and replaced with arithmetic (algebra, that is).

26 One should contrast this modern view with that of Isaac Newton, who expressed the following in his "Lectiones opticae" held in Cambridge around 1670: "We must indeed become philosophizing geometers and philosophers knowing to apply geometry, if we want to gain knowledge of Nature based on evident truths, instead of contenting ourselves with the conjectures and probabilities spread everywhere" (my transl. from Newton's Latin).

27 In my book "Die Rehabilitierung des Galileo Galilei" (Sankt Augustin 2007), which basically was meant to show that Galileo was right when he claimed to have proved the motion of the Earth), I baptized analytical or "classical" mechanics anew, as "Berliner Mechanik" (the "Berlin Mechanics"), because it was essentially conceived in Berlin, at the Prussian Academy of Sciences, where the most effective promoters of this science, Euler and Lagrange, both worked together for years in the middle of the 18th century.

28 It should be mentioned here that the notion "dynamics" as well as the notion "energy" is of Aristotelian origin. Both were introduced into the new theory of motion by G. W. Leibniz. As a matter of fact, the Leibnizian theory of process as an exchange of "states of energy" (e.g. potential to kinetic), or state transition, has very much to do with the Aristotelian concepts of *dynamis-energeia-entelecheia* and their relation to each other.

29 Here the physician Julius Robert Mayer (1814-1878) must be cited among the founders of thermodynamics, James Prescott Joule (1818-1889) and Rudolf Clausius (1822-1888). Mayer's paper of 1841 "Bemerkungen über die Kräfte der unbe-lebten Natur" (*Remarks on the forces of inanimate Nature*) shows how intimately related his views were to the science and philosophy of G.W. Leibniz. Here is a quote from

Mayer (my transl.): “Forces are causes; therefore the principle *causa aequat effectum* can perfectly be applied to them. If the cause c has the effect e, then c = e.” This view may to some extent also be the consequence of a wide-spread confusion as to Newton’s second and third law of motion, the second one stating a “proportionality of cause and effect”, the third one stating an “equality of action and reaction”. Some erroneously take “action” as being tantamount to “force”, or “cause”, or “energy” (cf. Wolfgang Hofkirchner, The Hidden Ontology, Emergence 3 (3) 2001 p. 7),. The confusion even extends to the German notion “Wirkung” (“effect”) that is used to denote Planck’s quantum of “action”, $E = h\nu$, arbitrarily changing the meaning of E from cause to effect.

30 Actually thermodynamics does imply a “causal law of motion”, namely the proportionality between pressure p (= “force”, or “energy”) and temperature T , respectively nT (= “motion” analogous to the concept of “momentum”, mv), according to the formula $p = nT \times k$ (n = 1, 2, 3; k = Boltzmann’s constant, the “proportionality constant”); note that this formula is also an equivalent of the thermic state equation called “ideal gas law”.

31 I quote this from Erich Truckenbrodt, Fluidmechanik vol. 1, Berlin 1980, p. 2 (my transl. from the German).

32 I rely on Max Jammer, Concepts of Force, Cambridge Mass., 1957, p. 161, fn. 12. If this is true, it gives a remarkable example for the manner of some scientists to manipulate mathematical formulae at will, or, say, “for the sake of mathematical convenience” (so did Coriolis according to Jammer). I find another such example in Roberto Torretti, The Philosophy of Physics, Cambridge, 1999, p. 47. This author, after having introduced Newton’s second law correctly according to “$\Delta\mathbf{p} \propto \mathbf{F}$”, thinks it “more reasonable to put $\mathbf{F} = \mathbf{f}\Delta t$” and so to develop the *different* formula “$\mathbf{f} \propto \Delta\mathbf{p}/\Delta t$ “ which he next, arbitrarily replacing the “$\propto$” with “=”, renders into $\mathbf{f} = \Delta\mathbf{p}/\Delta t$ (force equals mass-acceleration, that is) “by a good choice of units”, as he says it. Note, however, that this “good choice” arbitrarily again *presupposes* **f** and $\Delta\mathbf{p}/\Delta t$ to be *homogenous entities* (entities having equal dimensions, that is, which dimensions accordingly reduce), so that their quotient is prepared to result in a *dimensionless number* that can be omitted. – Which logical analysis proves Torretti’s argument circular and worthless.

33 Every careful reader of Galileo’s and Newton’s works will admit that their theories were conceived and must be understood as “quantum theories” of light and matter. Cf. Fritz Bopp, Newtons Optik als unvollendetes quantenphysikalisches Konzept, Phys. Bl. 40 (1984) Nr. 9 S. 306; and: Newtons Wissenschaftslehre als Basis der Quantenphysik, Ann. d. Phys. 7. Folge Bd. 42 Heft 3, 1985, S. 217.

34 See footnote 13.

35 Leibniz’s concept of the measure of “force of motion” is synonymous with the concept of energy of classical mechanics and thermodynamics. To see it as a misconception as it implies the absurdity of a moving body to exist at different places in space at the same time requires a *geometric* analysis, which is shortly sketched in section VI of this paper. Cf. also footnote 37 for the difference between classical and quantum mechanical Energy concepts.

36 Peter Guthrie Tait to the best of my knowledge was the only one to ever realize Newton’s explanation of the term “action” as something similar to the concept of “energy”. See P.G. Tait, On the Conservation of Energy, Phil. Mag. 25 (4). 1863, p. 429-443. I cited Tait in my second paper to be published: Experimental Philosophy Reappraised, Sp. Sci. Techn. Vol. 9 nr. 2 (1986); footnote 15 to the entry nr. 2 on my website.

37 Cf. Shufeng Zhang, Entropy: A concept that is not a physical quantity; Physics Essays 25 (2), 172-176, 2012. All this corroborates my view that the “classical” scalar, “squared” energy term means something different from energy in quantum mechanics. I refer to the quantum mechanical term E = p × c. Here we have energy E and momentum p (= mv) in a *linear* relation to each other, and as *vector quantities*. Note that the product p × c of the variable vector v (velocity), and the constant, c (“vacuum velocity of light”), will never yield *a scalar square of the variable*, as it is the case in classical energy E = mv^2.

38 For this formulation of the laws of thermodynamics I refer to an email of Malcolm Dean, dated 5 July 2012.

39 See my website, www.neutonus-reformatus.com, entry nr. 39.

40 Cf. Ed Dellian, On cause and effect in quantum mechanics, Spec. Sci. Techn. Vol 12 nr. 1 (1989) p. 45; (entry nr. 7 on my website).

41 Note however that others such as Stephen Hawking simply use “proportional” synonymously to “equivalent”; Stephen Hawking, A Brief History of Time, London 1988. See the Bantam ed. 1992, p. 21: “the **equivalence** of energy and mass”; p. 114: “energy is **proportional to** mass” (my emphasis).

42 Cf. John von Neumann, Mathematische Grundlagen der Quantenphysik, Berlin 1932, Introduction, p. 5-15.

43 Werner Heisenberg, Physikalische Prinzipien der Quantentheorie, Stuttgart 1958, p. 93 („Partikelbild der Strahlung“; i.e. *Particle picture of radiation*).

44 E. Wigner, in: Communications on pure and applied mathematics 13 no. 1 (1960).

45 Roger Penrose, The Road to Reality – A Complete Guide to the Laws of the Universe, New York 2005.

46 Robert Rosen, Life Itself, A Comprehensive Inquiry Into the Nature, Origin, and Fabrication of Life, New York 1991.

47 Adrian Bejan and J. Peder Zane**,** Design in Nature; How the **Constructal Law** Governs Evolution in Biology, Physics, Technology, and Social Organization, New York 2012.

48 Galileo in his “Two new Sciences”, Second Day, has Simplicio saying: “Truly I begin to understand that although logic is a very excellent instrument to govern our reasoning, it does not compare with the sharpness of geometry in awakening the mind to discovery.” Sagredo answers: “It seems to me that logic teaches how to know whether or not reasonings and demonstrations already discovered are conclusive, but I do not believe that it teaches how to find conclusive reasonings and demonstrations” (the Stillman Drake ed., p. 133).

49 It has already been noticed that this result disproves the “Big Bang” hypothesis, as it disproves Hubble’s basic assumption of a velocity-distance proportionality of galaxies receding from the observer; see my website www.neutonus-reformatus.com, entry nr. 37.

50 I add these short philosophical remarks here only to indicate in which direction the still missing answer to the question of “meaning” of modern science might be found. But to go into detail about this topic is not my aim here; therefore at the moment I want to leave any further considerations to the reader.